for noncarry commodities. Basically his model states that the supply and demand varies over time for short hedges and for long hedges. Any risk premium (the difference between the forward price and the expected spot price on the delivery date) will be a function of the net hedging activity occurring at any moment, and it can change in sign (from backwardation to contango) over the life of the contract.

Thus, if risk-averse hedgers are currently attempting to manage their risk exposures by actively selling forward contracts, the forward price quoted by dealers will decline. This will occur in one of two ways. First, if the dealers are playing the role of speculators, we conclude that they will not be willing to accept the price risk unless there is a risk premium, $F_t < E(\tilde{S}_T)$. Alternatively, the dealers will use other derivatives markets to shed themselves of the price risk they assumed by buying forwards from the hedgers. They will do this by selling futures, or by using options or swaps. But now, *their* hedging activities will suppress prices in those markets, and there will be a resulting secondary impact on forward prices. But either way, we conclude that active short hedging activity will lead to lower forward prices, which are likely to be less than the future spot prices expected by market participants.

It seems naive to believe that hedgers are consistently net short. Users of commodities hedge their planned future purchases of raw materials, too, and they will tend to buy forward contracts in those situations. Thus, like Cootner, we might expect to observe backwardation and contango situations at different times for different commodities. The question of whether forward prices equal expected future spot prices is ultimately an empirical one, and many researchers have examined forward prices for the existence (if any) of a risk premium that would answer whether the unbiased expectations hypothesis holds, or whether backwardation or contango exists.

Kamara (1984, p. 70) summarizes the many studies that test whether forward prices are the unbiased expectation of future spot prices writing about futures markets by stating:

> In sum, although it is widely accepted that futures markets are used by risk-averse hedgers, the evidence suggests that hedgers have been able to purchase the insurance very cheaply. As a result, forward prices on average do not contain a significant risk premium.

5.1.8 Valuing a Forward Contract After Origination

After a forward contract has been originated, forward prices change. In other words, on March 1, you might agree to buy an ounce of gold on October 1, at the forward price of \$320/oz. One month later, on April 1, the forward price for delivery of gold on October 1 might be \$330/oz. The value of your original forward contract (that obligates you to buy gold at \$320/oz.) is merely the present value of the difference between the two forward prices.

Thus, define

$F(0,T)$ = forward price of the original contract, created at time 0, for delivery of the underlying asset at time T

$F(t,T)$ = forward price at time t, for deliver also at time T

$h(t,T)$ = spot interest rate that exists at time t, for borrowing and lending until time t with $h(t,T) = r(T-t)/365$, where r is the annual interest rate for borrowing and lending over $(T-t)$ days.

Then, the value of the original forward contract, at time t, is

$$\frac{F(t,T) - F(0,T)}{1 + h(t,T)}$$

If the forward price has risen $[F(t, T) > F(0, T)]$, the value is positive for the long position (an asset) and negative for the short position (a liability). If the forward price has declined $[F(t, T) < F(0, T)]$, the value is positive for the individual who is short the forward contract and negative for the individual who is long the contract.

5.2 Forward Exchange Rates

5.2.1 The Standard Approach to Deriving the Forward Exchange Rate

The standard cost-of-carry pricing formula, with one small modification, determines foreign exchange forward prices and forward prices. Define:

N = number of units of foreign exchange in one forward contract

T = number of days until delivery

r_d, r_f = annual domestic and foreign interest rates, respectively

$h_d(0, T)$ = domestic unannualized interest rate from today until the delivery date = $(r_d)(T)/365$

$h_f(0, T)$ = foreign unannualized interest rate from today until the delivery date = $(r_f)(T)/365$

When performing the cash-and-carry arbitrage trades, borrow funds to purchase $N/[1+h_f(0, T)]$ units of foreign exchange and sell a forward contract for the delivery of N units. In other words, buy the present value of N units of foreign exchange, where the interest rate used in determining the present value factor is the *foreign* interest rate. Thus, if the cost-of-carry model assumes no carry return,[14] the forward exchange pricing model specifies

$$F = \frac{S}{1 + h_f(0, T)}[1 + h_d(0, T)] = \frac{S[1 + h_d(0, T)]}{1 + h_f(0, T)} \tag{5.6}$$

Equation (5.6) is the standard cost-of-carry forward pricing model when there is no carry return, except $S/[1 + h_f(0, T)]$ is substituted for S.

When performing cash-and-carry arbitrage,[15] the arbitrageur borrows funds to purchase the present value of N units of foreign exchange and invests those units to earn the foreign riskless interest rate. On the delivery date, the arbitrageur will have N units of the foreign currency to deliver, thereby allowing the requirements of the short forward position to be fulfilled.

Equation (5.6) can be rearranged and reexpressed in terms of annual interest rates as follows:

$$\frac{F}{S} = \frac{\left[r_d(T/365)\right] + 1}{\left[r_f(T/365)\right] + 1} = \frac{365 + r_d T}{365 + r_f T} \tag{5.7}$$

EXAMPLE 5.4 Suppose that the delivery date for forward delivery of euros is 144 days hence. The annualized domestic (U.S.) interest rate is 7% if an individual wishes to borrow or lend for 144 days. The euro interest rate is 4.5%/year. The spot exchange rate is \$0.95/€. Determine the theoretical forward price.

Use Equation (5.6) to obtain the solution:

$$F = \frac{S[1 + h_d(0,T)]}{1 + h_f(0,T)} = \frac{0.95[1 + (0.07)(144)/365]}{1 + (0.045)(144)/365} = \frac{0.95(1.027616)}{1.017753} = \$0.9592/€$$

The theoretical forward price is \$0.9592/€. By going long one euro forward contract, a trader locks in the purchase price of \$0.9592/€. Figure 5.7 shows how FinancialCAD uses the function aaFXfwd to solve the problem. The function aaAccrual_days (also shown in Figure 5.7) is used to verify that there are 144 days between September 24, 1999 and February 15, 2000.

AaFXfwd		
FX spot - domestic / foreign	0.95	
Rate – domestic - annual	0.07	
Rate – foreign - annual	0.045	
Value (settlement) date	24-Sep-2000	
Forward delivery or repurchase date	15-Feb-2001	
Accrual method - domestic rate	1	actual/365 (fixed)
Accrual method - foreign rate	1	actual/365 (fixed)
Statistic	2	fair value of forward (domestic /foreign)

fair value of forward (domestic /foreign)	**0.959206418**

AaAccrual_days		
Effective date	24-Sep-2000	
Terminating date	15-Feb-2001	
Accrual method	2	actual/360

Number of business days from an effective date to a terminating date	**144**

Figure 5.7 The FinancialCAD function aaFXfwd is used to compute the theoretical forward exchange price. The time from an effective date to a terminating date is found by using a AaAccrual_days.

The FinancialCAD function aaFXfwd_sim allows you to observe how the forward price varies as a function of either the spot rate, the domestic interest rate, or the foreign interest rate. You choose the independent variable on the line titled x-axis. The program's output table (Figure 5.8a) was plugged into the Excel Chartwizard to produce a graph (Figure 5.8b).

Now, suppose that the actual 144-day forward price is \$0.953/€. Because this price is less than the 144-day theoretical forward price of 0.9592, arbitrageurs will engage in reverse cash-and-carry arbitrage. Assume that N = €125,000 An arbitrageur would perform the following series of reverse cash-and-carry trades:

Today

- Borrow 125,000/1.017753 = €122,819.58 at the foreign interest rate of 4.5%/year. Sell the €122,819.50 at the spot price of \$0.95/€, and receive \$116,678.525.
- Lend the proceeds of \$116,678.525 for 144 days at the domestic interest rate of 7%/year.
- Go long one forward contract that requires the purchase of €125,000 at the forward price of \$0.953/€.

  Zero cash flow today

At Delivery (144 days hence)

- Receive the loan proceeds of \$116,678.525(1.02762) = \$119,901.186.
- Fulfill the requirements of the forward contract: by purchasing €125,000 at the contracted price of \$0.953/€. The total purchase price is \$119,125.
- Repay the euros borrowed. With interest, the obligation is €125,000.

  Cash inflow at delivery = \$119,901.186 − \$119,125 = \$776.186

Note two points. First, the arbitrage profit of \$776.186 is realized with no initial capital outlay. Second, if arbitrage opportunities such as this ever emerged, individuals would sell euros in the spot market and go long forward contracts. The process of arbitrage would lead to a lower spot euro price and a higher forward euro price. Arbitrageurs' trades will correct the initial mispricing.

Data on foreign interest rates can be found at several websites, such as www.bloomberg.com/markets/iyc.html (Bloomberg's site). Charts that depict how foreign interest rates have moved in the past few years can be found at www.yardeni.com/finmkts.html (Ed Yardeni's site). The *Wall Street Journal* gives yields on international government bonds, which are long-term foreign interest rates, in its "Bond Market Data Bank" column; an example is given in Figure 5.9. *Barron's* provides a graph of short-term interest rates for British pounds, euros, and Japanese yen, and a graph of yields for long-term British, German, and Japanese government bonds in its weekly "Current Yield" column; an example is presented in Figure 5.10.

5.2.2 An Alternative Derivation of the Theoretical Forward Price

Assume that you have one dollar to invest for T days. You have two ways to invest the money:

Method 1: Invest the dollar in the United States. At the end of the T days, you will have $1[1 + h_d(0, T)] = 1 + (r_d T/365)$.

(a)

AaFXfwd_sim		
FX spot - domestic / foreign	0.95	
Rate - domestic - annual	0.07	
Rate - foreign - annual	0.045	
Value (settlement) date	24-Sep-99	
Forward delivery or repurchase date	15-Feb-00	
Accrual method - domestic rate	1	actual/ 365 (fixed)
Accrual method - foreign rate	1	actual/ 365 (fixed)
Statistic	2	fair value of forward (domestic / foreign)
x-axis	1	domestic spot price
Simulation range x-axis	0.01	
Orientation	1	vertical format

Simulation table - aaFXfwd_sim

Items in X axis simulation range	items in Y axis simulation range
0.9405	0.949614353
0.941857143	0.950984648
0.943214286	0.952354943
0.944571429	0.953725238
0.945928571	0.955095533
0.947285714	0.956465828
0.948642857	0.957836123
0.95	0.959206418
0.951357143	0.960576712
0.952714286	0.961947007
0.954071429	0.963317302
0.955428571	0.964687597
0.956785714	0.966057892
0.958142857	0.967428187
0.9595	0.968798482

(b)

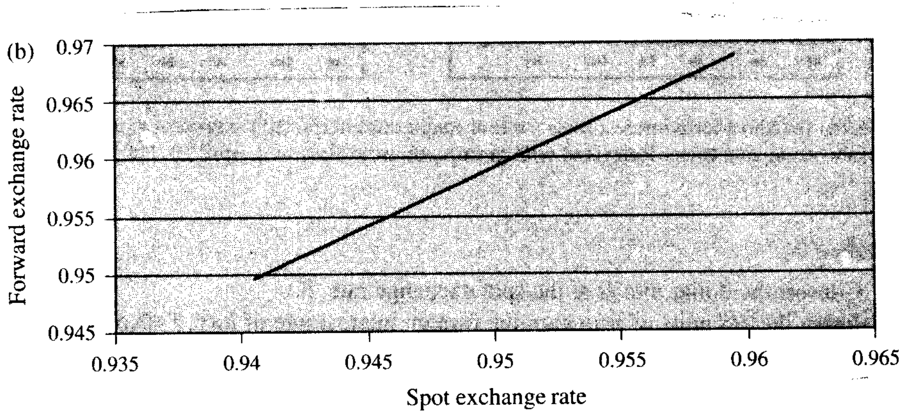


Figure 5.8 (a) The FinancialCAD function aaFXfwd_sim computes a table of theoretical forward exchange prices as a function of either the spot exchange rate, the domestic interest rate, or the foreign interest rate. (b) Forward exchange rate as a function of the spot exchange rate.

BOND MARKET DATA BANK 6/4/01

INTERNATIONAL GOVERNMENT BONDS

COUPON	MATURITY (Mo./YR.)	PRICE	CHANGE	YIELD*	COUPON	MATURITY (Mo./YR.)	PRICE	CHANGE	YIELD
Japan (3 p.m. Tokyo)					**Germany** (5 p.m. London)				
4.50%	06/03	109.03	− 0.01	0.06%	3.25%	02/04	97.23	+ 0.06	4.354%
3.40	06/05	112.50	− 0.03	0.27	5.00	05/05	101.66	− 0.01	4.530
1.30	06/11	100.53	+ 0.04	1.24	5.00	07/11	99.45	+ 0.00	5.069
1.90	03/21	100.07	− 0.22	1.90	5.50	01/31	98.09	+ 0.02	5.683
United Kingdom (5 p.m. London)					**Canada** (3 p.m. Eastern Time)				
7.00%	06/02	101.86	− 0.05	5.080%	6.00%	12/02	101.86	+ 0.04	4.698%
9.50	04/05	114.46	− 0.07	5.312	5.25	09/03	100.56	+ 0.07	4.986
6.25	11/10	108.19	− 0.03	5.146	6.00	06/08	102.06	+ 0.29	5.641
4.25	06/32	88.71	− 0.55	4.968	8.00	06/27	126.97	+ 0.61	5.949

*Equivalent to semi-annual compounded yields to maturity

Figure 5.9 Yields on government bonds from Japan, the United Kingdom, Germany, and Canada. (Reprinted with permission of *The Wall Street Journal*, page C20. © June 4, 2001.)

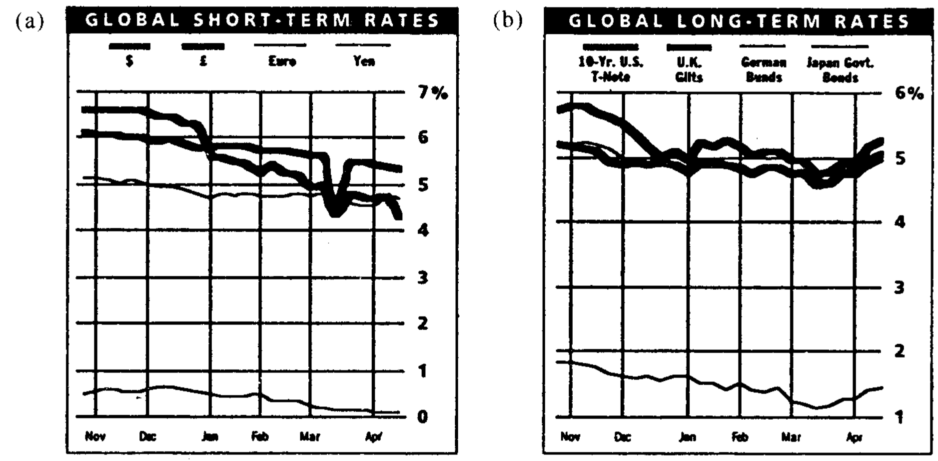


Figure 5.10 (a) Short-term interest rates for four major currencies. (b) Long-term yields for government bonds of the same countries. (Reprinted with permission from *Barron's*, April 23, 2001, page MW16.)

Method 2:

1. Convert the dollar into *fx* at the spot exchange rate, S.
2. Invest the $1/S$ units of *fx* to earn the foreign interest rate of $h_f(0, T)\%$ over T days.
3. Sell a forward contract that obligates you to deliver the $(1/S)[1 + h_f(0, T)]$ units of *fx* on the delivery date at time T. The forward price is F (expressed as the dollar price of a unit of foreign exchange *fx*).
4. At time T, your foreign investment will be worth $(1/S)[1+h_f(0, T)]$ units of *fx*. Under the terms of the forward contract, each *fx* unit can be converted to dollars at the exchange rate F. Thus, you will receive $(F/S)[1+h_f(0, T)]$ dollars.

The two methods are equivalent because they are both riskless lending transactions for T days. Therefore, you should have the same wealth at time T either way you invest. This means that

$$\$1[1 + h_d(0,T)] = \$1(F / S)[1 + h_f(0,T)]$$

which is the same as Equations (5.6) and (5.7).

If one strategy dominated the other, then investors would invest in the better strategy. For example, if method 1 provided a higher return than method 2, Americans would keep their dollars as dollars, and invest in the United States. Also, foreigners would sell their units of *fx* for dollars, lend them in the United States, and buy forward contracts on their *fx*.

5.2.3 The Implied Repo Rate

If the present value (found by using the foreign interest rate) of spot foreign exchange is purchased, and a foreign exchange forward contract is sold, this portfolio creates a long position in a synthetic T-bill. The rate of return earned on this synthetic T-bill is called the implied repo rate. If the implied repo rate exceeds your borrowing rate, cash-and-carry arbitrage is possible.

To find the unannualized implied repo rate, solve Equation (5.6) for $h_d(0, T)$. To compute the annualized implied repo rate when using an actual/365 day count method, solve Equation (5.7) for r_d.

$$\text{unannualized implied repo rate} = h_d(0,T) = \frac{F[1 + h_f(0,T)] - S}{S} \tag{5.8}$$

$$\text{annualized implied repo rate} = r_d = \frac{365F - 365S + FTr_f}{TS} \tag{5.9}$$

EXAMPLE 5.5 Assume that you can lend British pounds for three months in Great Britain at the rate of 6%/year. All day count methods are assumed to be 30/360. A long position in a synthetic (U.S.) T-bill is created by purchasing 62,500/[1 + (0.06/4)] = £61,576.35 and selling a forward contract requiring the delivery of £62,500 three months hence. At the spot exchange rate of \$1.64/£, the purchase of £61,576.35 will cost \$100,985.22. The pounds are lent in Great Britain, and a forward contract is sold at the forward price of \$1.65/£, which locks in a selling price. Three months later, the £62,500 is delivered, satisfying the terms of the forward contract, and (62,500)(1.65) = \$103,125 is received. The implied repo rate is (\$103,125 − \$100,985.22)/\$100,985.22 = 2.119% over three months, or 8.476%/year. If an individual can borrow in the United States for three months at an annual rate of less than 8.476%, and if transactions costs are ignored, then there is an arbitrage opportunity. Figure 5.11 shows how Financial-CAD would use the function aaFXfwd_repo_d to solve the problem. Note the use of the 30/360 accrual methods for both interest rates.

AaFXfwd_repo_d		
FX spot - domestic / foreign	1.64	
FX forward price - domestic / foreign	1.65	
Rate - foreign - annual	0.06	
Value (settlement) date	24-Sep-99	
Forward delivery or repurchase date	24-Dec-99	
Accrual method - domestic rate	4	30/ 360
Accrual method - foreign rate	4	30/ 360
Domestic repo rate	0.084756098	

Figure 5.11 FinancialCAD function aaFXfwd_repo_d computes the theoretical domestic repo rate obtained from buying a foreign currency and selling it forward.

In the example just presented, a 30/360 day count method is used, and thus Equation (5.9), the formula for the annualized implied repo rate, must be modified as follows:

$$r_d = \frac{360(1.65) - 360(1.64) + (1.65)(90)(0.06)}{(90)(1.64)} = 0.08476$$

5.3 Forward Interest Rates

5.3.1 Spot Rates and Forward Rates

A **spot rate** is an interest rate that exists today. If today is denoted time 0, then $r(0, t1)$ is denoted today's spot rate for a debt instrument that matures at time $t1$. Also, r denotes annual interest rates. Thus, $r(0, 3)$ is the spot three-year interest rate, expressed as an annual rate.[16]

To compute a **forward rate**, you need two spot rates for two different annual maturities, $t1$ and $t2$, where $t2 > t1$. Then, the forward rate can be computed by solving for $fr(t1, t2)$ in the following formula:

$$(1 + r(0, t2))^{t2} = [1 + r(0, t1)]^{t1}[1 + fr(t1, t2)]^{t2-t1} \tag{5.10}$$

In a sense, the forward rate is an interest rate that exists in the future. That is, $fr(t1, t2)$ is the interest rate on a loan beginning at time $t1$ and ending at time $t2$. Note that fr is an annual forward rate. Thus, in Equation (5.10), all times such as $t1$ and $t2$, are expressed in years.

EXAMPLE 5.6 The spot rate for a six-year debt instrument is $r(0, 6) = 12\%$, and the spot rate for a two-year debt instrument is $r(0, 2) = 10\%$. Find the forward rate for a debt instrument that begins two years hence and ends six years hence. That is, find the rate quoted for a 24×72 FRA.

Use Equation (5.10) to solve for the forward rate, $fr(2, 6)$:

$$(1.12)^6 = (1.10)^2[1 + fr(2,6)]^4$$

$$1.9738227 = 1.21[1 + fr(2,6)]^4$$

$$(1.6312584)^{0.25} = [1 + fr(2,6)]$$

$$1.130136 = [1 + fr(2,6)]$$

$$fr(2,6) = 13.0136\%$$

When working with maturities of less than one year, another method of computing forward rates often is more useful. Define $h(0, t2)$ and $h(0, t1)$ as today's unannualized spot interest rates for debt instruments that mature at times $t2$ and $t1$, respectively. Define $fh(t1, t2)$ as the forward unannualized rate for a debt instrument that begins at time $t1$ and ends at time $t2$. Then the following formula can be used to solve for the forward rate:

$$1 + h(0,t2) = [1 + h(0,t1)][1 + fh(t1,t2)] \tag{5.11}$$

Note that in Equation (5.11), there are no exponents. Also, note that the spot and forward rates are *unannualized.*

EXAMPLE 5.7 A \$10,000 face value Treasury bill that matures in 34 days is priced at \$9907.71, while the price of a T-bill that matures in 55 days is \$9836.95. Find the forward rate from day 34 to day 55.

To solve, first, compute the unannualized spot rates. Given $P = 9907.71$, $F = 10{,}000$, and $T = 1$ period (of 34 days), then $10{,}000 = 9907.71\ (1 + h)$, and $h(0, 34) = 0.9315\%$. By the same logic, the unannualized 55-day spot rate is $h(0, 55) = 1.65753\%$. Equation (5.11) is then used to find the unannualized forward rate, $fh(34, 55)$:

$$1.0165753 = 1.009315[1 + fh(34,55)]$$

$$fh(34,55) = 0.71933\%$$

There are two ways to annualize this rate. If 0.0071933 is multiplied by the number of 21-day periods in a year (there are 21 days from $t1 = 34$ to $t2 = 55$), then the rate is annualized assuming simple interest:

$$0.0071933 \times 365/21 = 12.50256\%$$

Alternatively, the rate of 0.71933% can be compounded to obtain an annualized rate:

$$(1.0071933)^{365/21} - 1 = (1.0071933)^{17.380952} - 1 = 13.26713\%$$

In the case of compounding, interest earns interest every 21 days.

Because the yields on short-term money market instruments are computed in different ways (e.g., different securities use different day count conventions), and because there are different ways to annualize the unannualized yield, it is usually safer and less ambiguous to just deal with prices and unannualized yields, which is what Equation (5.11) does.

Figure 5.12 illustrates how the FinancialCAD function aaFRAi can be used to compute forward rates. It cannot be used to compute rates when they are compounded. Thus, the software cannot be used to compute $fr(2, 6)$ as was done for the four-year interest rate that will exist in two years. The figure presents the solution to the Treasury bill problem when the simple interest approach is used to annualize $fh(34, 55)$. Note that discount factors must be entered, not interest rates. The entry on the line titled "FRA contract rate" is not used when the desired output statistic is the implied forward rate.

The FinancialCAD utility function aaConvertR_DFcrv will convert interest rates to discount factors. The example is shown in Figure 5.13. The October 28, 1999, yield to maturity is computed

aaFRAi		
Value (settlement) date	24-Sep-99	
Effective date	28-Oct-99	
Terminating date	18-Nov-99	
FRA contract rate	0.035	
Notional principal amount	1000000	
Accrual method	1	actual/ 365 (fixed)
Discount factor curve	t_43_1	
Interpolation method	1	linear
Statistic	2	implied forward rate

t_43_1
discount factor curve

grid date	discount factor	
24-Sep-99	1	
28-Oct-99	0.990770968	(=1/1.009315)
18-Nov-99	0.983694961	(=1/1.0165753)

implied forward rate	**0.125026309**

Figure 5.12 FinancialCAD function aaFRAi computes the theoretical forward rate given two spot rates. To use the function, the spot rates must be converted to discount factors.

aaConvertR_DFcrv		
value (settlement) date	24-Sep-99	
rate curve	t_48_4	
rate quotation basis	1	annual compounding
accrual method of rate	1	actual/365 (fixed)

t_48_4
holding cost curve

maturity date	yield to maturity
28-Oct-1999	0.104658291
18-Nov-1999	0.11527138

discount factor curve - aaConvertR_DFcrv

grid date	discount factor
24-Sep-1999	**1**
28-Oct-1999	**0.990771**
18-Nov-1999	**0.983695**

Figure 5.13 The FinancialCAD function aaConvertR_DFcrv converts interest rates to a set of discount factors.

by $P = 9907.71$, $F = 10{,}000$, and $T = 34/365 = 0.093150685$ year. Upon entering these values into your calculator, you should find that the yield solution is 10.4658291%. The November 18, 1999, yield is computed by $P = \$9836.95$, $F = 10{,}000$, and $T = 55/265 = 0.150684932$ year.

One last useful method of obtaining forward yields requires the use of prices. Define the price of a zero-coupon, \$1 face value debt instrument that matures at time $t1$ as $P(0, t1)$. Similarly, the price of a pure discount debt instrument that matures to be worth \$1 at time $t2$ is $P(0, t2)$. Then, the forward price of a \$1 face value debt instrument from time $t1$ to time $t2$, $FP(t1, t2)$, is found by

$$P(0, t2) = P(0, t1)FP(t1, t2) \tag{5.12}$$

Consider the example in which $r(0, 6) = 12\%$ and $r(0, 2) = 10\%$. We know that $P = F(1 + r(0, t))^{-t}$. Thus, $P(0, 6) = 1(1.12)^{-6} = 0.5066311$, and $P(0, 2) = 1/(1.10)^2 = 0.8264463$. Then, by Equation (5.12), we have

$$0.5066311 = 0.8264463FP(2, 6)$$

$$FP(2, 6) = 0.6130236$$

Thus, a debt instrument that will be issued two years hence and will be worth \$1 six years from today, will sell for \$0.6130236 two years from today. The annual rate of return on an investment of $P = \$0.6130236$ that will be worth $F = \$1$ four years later ($T = 4$) is found by solving for the interest rate x in the equation

$$F = P(1 + x)^T$$

$$1 = 0.6130236(1 + x)^4$$

and we find that $x = 13.0136\%$. This is exactly what we earlier found for $fr(2, 6)$.[17]

As another example that uses securities with maturities of less than a year, consider the problem with \$1 face value Treasury bills, where $P(0,34)=0.990771$ and $P(0,55)=0.983695$. Then the forward price, $FP(34,55)$ is:

$$(0.983695) = (0.990771)FP(34,55)$$

$$FP(34,35) = 0.9928581$$

What unannualized rate of return will an investor earn if \$0.9928581 is invested at time 34 and receives \$1 at time 55? The answer is $(F-P)/P=(1-0.9928581)/0.9928581=0.71933\%$. This is exactly what we found earlier for $fh(34, 55)$.

Thus, we have Equations (5.10)–(5.12), given earlier, three equivalent formulas to use when working with spot and forward annualized yields, unannualized yields, and prices:

$$[1+r(0,t2)]^{t2} = [1+r(0,t1)]^{t1}[1+fr(t1,t2)]^{t2-t1} \tag{5.10}$$

$$1+h(0,t2) = [1+h(0,t1)][1+fh(t1,t2)] \tag{5.11}$$

$$P(0,t2) = P(0,t1)FP(t1,t2) \tag{5.12}$$

5.3.2 How to Lock in a Forward Rate

Up until now, the forward rate has been just a number that is implicit in the term structure of (spot) interest rates. It turns out that an investor can actually lock in the forward rate as a borrowing rate or a lending rate! First, some assumptions must be made:

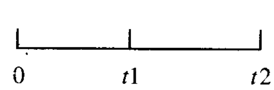
There are no transactions costs or bid–asked spreads.

Pure discount debt instruments of \$1 face value trade for any maturity.

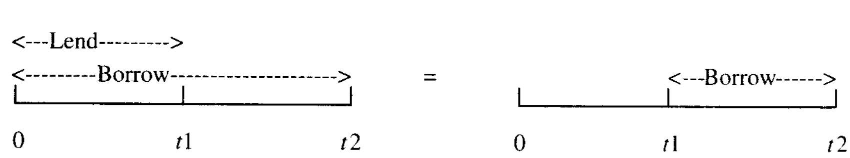
These securities can be bought or sold short.

Under these assumptions, any investor can lock in the forward rate as a borrowing rate or a lending rate, from time $t1$ to time $t2$.

Consider the following time line:



To lock in a borrowing rate from time $t1$ to time $t2$, borrow \$$X$ from time 0 to time $t2$, and lend \$$X$ from time 0 to time $t1$:



To lock in a lending rate from time $t1$ to time $t2$, lend \$$X$ from time 0 to time $t2$, and borrow \$$X$ from time 0 to time $t1$:

```
<---Borrow------>
<---------Lend----------------------->     =              <---Lend------>
|----------------|-------------------|          |----------------|--------------|
0               t1                  t2          0               t1             t2
```

An investor lends by purchasing debt instruments. An investor borrows by short selling debt instruments.

EXAMPLE 5.8 In Example 5.6 we specified that $r(0,6)=12\%$ and $r(0,2)=10\%$. We then computed the forward rate of $fr(2,6)=13.0136\%$. An investor can lock in 13.0136% as an annual borrowing rate from time 2 to time 6 by borrowing \$1 (or \$$X$) for six years at 12%/year and lending \$1 (or \$$X$) for two years at 10%/year. The resulting cash flows are as follows:

Time 0

Borrow \$1 for six years at 12%/year	+\$1
Lend \$1 for two years at 10%/year	<u>−\$1</u>
Net cash flow	0

Time 2

Get repaid on loan	$+1(1.1)^2=+\$1.21$

Time 6

Repay borrowed funds	$-1(1.12)^6=-\$1.9738227$

The result of both borrowing and lending today is that the investor has locked in a cash inflow at time 2 of \$1.21 and a cash outflow at time 6 of \$1.9738227. Thus, the investor has borrowed \$1.21 for four years at 13.0136%/year. Obviously, by adjusting the value of \$$X$, investors can borrow any amount from time 2 until time 6 at $r(2,6)=$ 13.0136%/year.

EXAMPLE 5.9 We can use the data from Example 5.7 to demonstrate that an investor can lock in a lending rate for a 21-day period, beginning 34 days hence. The unannualized rates are $h(0,34)=0.9315\%$ and $h(0,55)=1.65753\%$. The investor should lend \$1 for 55 days and borrow \$1 for 34 days. The resulting cash flows are as follows:

Time 0

Borrow \$1 for 34 days	+\$1
Lend \$1 for 55 days	<u>−\$1</u>
Net cash flow	0

34 days hence

Repay loan	− 1.009315

55 days hence

Get repaid on loan	+ 1.0165753

Thus, suppose a firm knows that 34 days from now, it will receive $750,000 and it knows that 55 days from now, it will have to make a large payment to a supplier. It would like to lock in the prevailing unannualized 21-day forward lending rate of 0.71933% during that time; after compounding, this *fh*(34, 55) is equal to 13.26713%/year, and the firm believes this is an attractive rate of return.

The firm would borrow $743,078.23 for 34 days, and lend the same amount for 55 days.[18] Then, in 34 days, it would repay its loan with interest, a cash outflow equaling $750,000. Twenty-one days after that, it would receive $755,394.97. The net result is that, at time 0, it has locked in the unannualized forward rate of 0.71933% on a 21-day loan of $750,000 made 34 days hence.

These illustrations of how to lock in forward borrowing and lending rates also provide "proofs" of how forward interest rates are derived using the cost-of-carry model. When we found the forward borrowing rate, what we did was sell short the underlying asset (the six-year debt instrument) and lend the proceeds for the period of the forward contract (two years). When we computed the forward lending rate, we borrowed for the period of the forward contract (34 days) to be able to buy the underlying asset (a debt instrument with 55 days to maturity). Thus, the same cost-of-carry logic used for finding forward exchange rates and forward commodity prices serves also in computing forward interest rates.

5.4 Summary

This chapter derived theoretical forward prices for commodities, foreign currencies (forward exchange rates), and interest rates (forward rates). The standard model for computing theoretical forward prices is the cost-of-carry model, $F = S + CC - CR$. Arbitrage ensures that forward prices conform to this model. If forward prices are too high, traders will borrow, buy the spot good, and sell the overpriced forward contract; this is called cash-and-carry arbitrage. If the forward price is too low, traders will sell the good in the spot market, lend the proceeds, and buy the cheap forward contract; this is called reverse cash-and-carry arbitrage.

The cost-of-carry model works very well for gold and for financial assets such as currencies, debt instruments, and stocks. However, the model sets only an upper pricing level for theoretical forward prices of all physical commodities except gold. What this means is that forward prices of commodities (agricultural goods such as wheat, energy products such as crude oil, etc.) will often be below their theoretical cost-of-carry values. The reason for this is that physical commodities offer a convenience yield. Users of these goods do not wish to sell them, or sell them short, because they need them as part of ongoing production processes.

Forward prices of financial assets, and of gold, do not equal the expected spot prices at delivery. However, forward prices of other physical commodities may equal expected future spot prices; the theory that predicts this is the unbiased expectations hypothesis of forward prices. However if risk-averse hedgers are actively selling forward contracts, the state of prices is called normal backwardation, and forward prices will be below what investors expect to prevail in the future. And if hedgers are actively buying forward, we are said to be in contango, and forward prices lie above expected future spot prices.

Cost-of-carry arbitrage establishes theoretical forward exchange rates. The arbitrageur must buy the present value of the needed units of foreign exchange, discounted at the foreign interest rate. Then, a forward contract is sold. Equation (5.6) is the theoretical forward exchange rate model.

Although the cost-of-carry model was not explicitly used to compute theoretical forward interest rates, the concept is very much present in the derivation of Equations (5.10)–(5.12). This should be apparent by a careful reading of Section 5.3.2.

Theoretically, forward prices and futures prices will be almost identical. In Chapter 6, we explain the differences between forward and futures contracts, and in Section 6.5 of that chapter we address the question of how daily resettlement of futures might create a difference between the prices of the two types of contract.

References

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Notes

[1]See Appendix A, at the end of this chapter, for more about short selling.

[2]$h(0, T)$ is the interest rate that applies to the period beginning at time 0 and ending at delivery, time T. If r is the annual interest rate and there are T days until delivery, then we find by using the actual/365 day count method that $h(0, T) = rT/365$. We assume that this interest, and the carry return, are both payable at time T. See Appendix B to this chapter for additional material on different day count methods.

[3]If the carry return cash inflows were received prior to time T, then CR should include the interest that can be earned on those cash flows. In other words, CR is actually the future value of the carry return cash flows. See Appendix C to this chapter for more discussion on computing the future value of the carry return cash flows.

[4]Problems 5.1 and 5.2d ask the student to apply the cost-of-carry formula to situations in which an individual gets to use only a fraction of the proceeds from the short sale.

[5]See Appendix B to this chapter for a discussion of different day count methods.

[6]The purchase of a good and the sale of a forward contract on that good creates what is called a **synthetic Treasury bill**. In a cash-and-carry arbitrage, the synthetic Treasury bill earns a rate of return in excess of spot Treasury bills. Hence, one borrows by selling spot Treasury bills and lends by buying synthetic Treasury bills.

[7]It is not unusual for the loan to be slightly below the securities' market value.

[8]In practice, the day count method for annualizing repo rates is the actual/360 approach.

[9] In addition, storage and carrying costs will also differ across different market participants, and they often change over time.

[10]It is possible that the supply of arbitrage capital is limited or faces competition from other trading strategies. If investors are overwhelmingly bullish, they may perceive greater benefits from buying forwards and futures even when these are overvalued. Similarly, hedgers will buy forward contracts even when their prices are too high, and sell them even when forward prices are below what the cost-of-carry pricing model predicts. If these situations do exist, arbitrage opportunities will also exist and may even persist for extended periods of time.

[11]Note that many of the concepts discussed in this chapter, like normal backwardation and contango, were initially developed in theories of futures pricing. However the theories that lead to these concepts are also applicable for forward prices.

[12]Arbitrageurs were not explicitly considered in Keynes's model.

[13]Many practitioners use the terms backwardation and contango when viewing the term structure of futures prices, vs the spot price. If futures prices are lower, the more distant the delivery date, then these individuals say that the market is in backwardation, or that the futures market is "inverted."

[14]Alternatively, the foreign interest earned by investing the foreign currency can be thought of as a carry return.

[15]When a currency is the underlying asset, this arbitrage is also known as **covered interest rate arbitrage**, and **covered interest parity**.

[16]The spot rates are the yields available on zero-coupon bonds.

[17]Alternatively, enter $PV = -0.6130236$, $FV = 1$, $N = 4$, and then $CPT\ i = ?$ into your financial calculator.

[18]Given that $T = 34$ days, and $h(0, 34) = 0.9315\%$, the present value of \$750,000, is \$743,078,23.

PROBLEMS

5.1 Assume that you are an individual who gets to use only a fraction, x, of the proceeds from the short sale of the good underlying a forward contract $0 \leq x \leq 1$. Otherwise, S is the price of the spot good, r is the annual interest rate, and CR is the future value of any carry return. What is the cost-of-carry pricing formula that determines the range (upper bound and lower bound) of prices for which you cannot arbitrage? (Problem 2d provides an application of this model).

5.2

a. Determine the no-arbitrage upper and lower forward price boundaries for a stock that is quoted at \$62.50/share (bid) and \$63.00 (asked). The stock does not pay any dividends. The delivery date of the forward contract is eight months hence. Your annual lending rate is 6%. Your annual borrowing rate is 7%. Use the FinancialCAD function aaCDF to verify your results (for both boundaries).

b. Suppose that $S=\$62.75$, $r=6\%$, and the stock will pay a \$0.40/share dividend next month, and quarterly thereafter (i.e., at $t=1$, $t=4$, and $t=7$). The interest rate is expected to remain constant at 6% for all maturities up to a year in length. Compute the theoretical forward price for the stock. Use aaEqty_fwd to verify your answer.

c. Assume all data from part **b** to be valid. If $F=63$, then compute the implied repo rate for the forward contract. Use aaCDF_repo to verify your answer. How would you arbitrage if $F=63$?

d. Now assume that $S=\$62.75$, $r=6\%$, and there are no dividends, but if you sell the stock short, you will get to use only 70% of the proceeds. Compute the theoretical upper and lower boundaries for the forward price for the stock. These are the relevant price boundaries within which *you* cannot arbitrage.

5.3

a. Spot gold sells for \$300/oz. You can risklessly borrow and lend any amount of money at an annual rate of 10%. What is the forward price for a contract that calls for the delivery of 100 oz. of gold 5 months from today? Use FinancialCAD to check your answer.

b. Suppose that $F_0=313$. Explain how you would arbitrage, given the data in part a. Present all the trades you would make today, and the trades that you would make five months hence. Are you performing cash-and-carry arbitrage, or reverse cash-and-carry arbitrage?

c. Given $F_0=313$, compute the implied repo rate for the forward contract. Use FinancialCAD to check your answer.

5.4

a. Spot gold sells for \$300/oz. Assume that you can borrow at an annual rate of 10% and risklessly lend any amount of money at an annual rate of 9%. What is the lowest forward price for that contract that will not allow arbitrage for you? What is the highest forward price? The forward contract calls for the delivery of 100 oz. of gold five months from today.

b. Now, in addition to the information in part a, assume that the bid price of gold is \$300/oz. and the asked price is \$301/oz. Compute the range of gold forward prices (the delivery date is still five months hence) that can exist without permitting you the opportunity to arbitrage. Use FinancialCAD to check your answers.

c. In addition to the data in part b, suppose that you face a \$50 commission on the forward contract that is paid upon offsetting your forward position. What are the lowest and highest forward prices that preclude arbitrage, now?

d. Finally, assume that you can short-sell gold, but you will receive only 10% of the proceeds from the short sale; the remainder stays on deposit with the broker. What is the lowest forward price for that contract that will not allow arbitrage for you? What is the highest forward price?

5.5 What is the definition of a risk premium for a forward contract? Define it in terms of net hedging and net speculation concepts (normal backwardation and contango).

5.6 State the differences among the following: repo rate, implied repo rate, and implied reverse repo rate.

5.7 On January 2, 1999, the spot price of West Texas Intermediate crude oil was $12.68/bbl. The crude oil forward prices for July 1999 delivery and January 2000 delivery were $12.57 and $10.43/bbl, respectively. What explains this "inverted market," in which the cash price exceeds the forward price and nearby forward price exceeds forward prices for even more distant delivery dates? What is the likely reason for arbitrageurs' failure to sell spot oil and buy forward contracts? If you expected the spot price of crude oil to be $12/bbl in January 2000, would you believe that the market was exhibiting normal backwardation or contango?

5.8

a. A stock sells for $125/share. It will trade ex-dividend two months hence, and the dividend amount will be $0.50/share. The interest rate is 5%. Compute the theoretical forward price for delivery four months from today. Use FinancialCAD to check your solution.

b. If the forward price was $125.40/share, explain how you would arbitrage. What is the implied repo rate of the forward contract? Use FinancialCAD to check your answer.

5.9

a. Today is March 14. The annual interest rate in Japan is 1%. The annual interest rate in Canada is 5%. The spot price of a Canadian dollar, expressed in terms of Japanese yen, is ¥86.25/Can$. Compute the theoretical forward price for Canadian currency for delivery on June 20 of the same year. Use FinancialCAD to check your answer.

b. Suppose that the actual forward price is ¥85.25/Can$. How would you arbitrage? Specify all the trades you would make on March 14 and on June 20. Compute the implied repo rate (in terms of Japanese interest rate). Use FinancialCAD to check your answer.

5.10 You have £4 million to invest. If you invest the money in Great Britain for two years, you can earn 5.8%/year. Alternatively, you can invest in France and earn 8%/year. The spot exchange rate is £0.10/FFR. The forward exchange rate for delivery two years hence is £0.098/FFR. Which method of investing do you prefer? Show your results.

5.11 The unannualized spot interest rates for different holding periods are as follows:

Period	Unannualized Interest Rate
1 month $h(0,1)$	0.01
2 months $h(0,2)$	0.0205
3 months $h(0,3)$	0.0305
4 months $h(0,4)$	0.041
5 months $h(0,5)$	0.052
6 months $h(0,6)$	0.063

Compute the unannualized forward rate from month 4 to month 6, $h(4,6)$. What is the annualized forward rate from month 4 to month 6? Use FinancialCAD to check your answer. Demonstrate how, by trading pure discount spot debt securities today, an investor can borrow $132,000 from month 4 until month 6 at the forward rate you computed.

5.12 If 10-year pure discount bonds are priced to yield 10%/year, and 14-year pure discount bonds are priced to yield 11%/year, at what interest rate can an investor borrow $100,000 from time 10 to time 14? Demonstrate how this can be done, showing the transactions and dollar amounts at all relevant dates.

Assume that all securities are infinitely divisible, that no transactions costs exist, and that investors get full use of the proceeds from short sales.

5.13 An investor wants to invest \$10 million in zero-coupon debt instruments maturing 10 years hence. She can invest in the United States and earn a 6.6% annual rate of return. Or, she can convert her \$10 million into Japanese yen and buy 10-year, zero-coupon, yen-denominated notes that are currently priced to yield 2.6%. The spot exchange rate is ¥120/\$.

a. At what forward exchange rate (for delivery 10 years hence) will the investor be indifferent between investing in the United States and in Japan? (Equivalently, consider if she chooses not to hedge. Then at what spot exchange rate that will exist 10 years hence will she "break even" in terms of an ex-post rate of return?)

b. What will be the investor's terminal wealth 10 years hence if she invests in the United States? Show that this equals her terminal dollar wealth if she invests in Japan instead.

5.14 Use current spot interest rate data to compute the forward interest rate for a five-month period commencing four months hence.

5.15 Use current spot interest rate data to compute the forward interest rate for a five-year period commencing four years hence.

5.16 Use current spot interest rate data and spot foreign exchange rate data to compute the theoretical forward price of a Canadian dollar for delivery one year hence.

5.17 On December 30, 1997, an interesting front page *Wall Street Journal* article analyzed why the Indonesian rupiah fell by 50% in value versus the U.S. dollar in the fall and winter of 1997. In discussing why Indonesian borrowers had borrowed enormous amounts of dollars, the article pointed out that the Indonesian government managed the dollar/rupiah exchange rate so that it declined at a rate of about 4–5% per year. "Indonesian borrowers faced a simple choice. They could borrow rupiah at 18 or 20% and not worry about the exchange rate. Or they could borrow dollars at 9% or 10% a year and then convert the proceeds to rupiah. A year later, they figured, it would take 4% or 5% more rupiah to buy the dollars needed to repay the loan, but the cost was less."

a. Explain why it was such a "simple choice" for Indonesians to borrow dollars. In particular, focus on the discussion in Section 5.2.2 of this chapter.

b. Explain what happened to the unhedged Indonesian borrowers of dollars when the dollar rose from about 2300 rupiah in July 1997 to about 5800 rupiah in December 1997.

c. How could have forward contracts been used to hedge Indonesian borrowers of dollars?

5.18

a. Today, $S=48$, where S is the price of the spot asset. If today's one-year interest rate, $r(0,1)$, is 6%, compute the theoretical forward price for delivery one year hence.

b. Three months later, $S=42$. The term structure of spot interest rates is as follows:

$$r(0,1/4) = 6\%$$

$$r(0,1/2) = 6.5\%$$

$$r(0,3/4) = 7\%$$

$$r(0,1) = 7.5\%$$

What is the value of the *original* forward contract (described in part a)? For which party is the forward contract an asset, and for which party is it a liability?

5.19 Suppose that on September 15, 1999, you sell €250,000 forward, for delivery on January 15, 2000. On September 15, 1999, the spot price of a euro is $1.05 (i.e., the spot exchange rate is $1.05/€), and the forward price for delivery four months later is $1.02/€.

On October 15, 1999, the following prices are observed:

Spot price	$1.03/€
Forward price for delivery three months hence	$1.04/€
Forward price for delivery four months hence	$1.06/€

Also, on October 15, 1999, the following interest rates are observed (in the United States):

For securities maturing one month later (i.e., on 11/15/99):	4%
For securities maturing three months later (on 1/15/00):	5%
For securities maturing four months later (on 2/15/00):	6%

a. Compute the value of your position as of October 15, 1999. This is equivalent to asking how much is exposed to default risk.

b. Is your position an asset or a liability? Explain why.

c. Are you more likely to want to default, or is your counterparty? Why?

5.20 Compute the forward exchange rate for delivery of euros 13 months hence. The spot exchange rate is ¥113.42/€. The interest rate on 13-month riskless debt instruments in Euroland is 5.25%. The interest rate on 13-month riskless debt instruments in Japan is 0.8%. Use FinancialCAD to check your solution.

5.21 On January 1 (today), a firm sells a 9× 12 FRA. The notional principal is $50 million. The contract rate is 8%. There are 365 days in a year. The following spot interest rates exist today (as shown on the first line of the accompanying table), and subsequently occur in the future (as shown on lines 2–5):

	Spot Interest Rates That Occur in the Future		
Date	**3-Month LIBOR**	**6-Month LIBOR**	**9-Month LIBOR**
January 1 (today)	7.4%	7.8%	7.9%
April 1	7.5%	8%	8.3%
July 1	8.2%	8.8%	9.1%
October 1	8.6%	9.4%	9.5%
January 1 (next year)	9.2%	9.6%	9.8%

a. If today's 9×12 FRA contract rate of 8% is theoretically correct, compute today's spot 12-month LIBOR. Use FinancialCAD to solve the problem.

b. When will the FRA be settled? (i.e., when will money be exchanged?)

c. Will the firm receive money (a profit) or have to pay money (a loss) on the settlement day? Why?

d. How much will be received or paid at settlement?

5.22 Use the FinancialCAD function aaCDF to compute the theoretical value of a futures contract on gold. The spot price of gold is $295/oz. The delivery day for the futures contract is five months from today. The interest rate is 6%. Assume no storage cost or convenience value. What happens to the theoretical fair value of the futures price if there is a small storage cost of 0.1? Explain why the theoretical futures price changes in that direction when there is a cost for storing the physical commodity.

5.23 Get a recent *Wall Street Journal* and find the column titled "Currency Trading" (its often on the foreign exchange page of Section C). Find and record the spot price of a dollar for Japanese citizens. Also find and record the forward price of a dollar for delivery six months hence. Then use aaFXfwd to compare the theoretical forward price of the dollar to the actual forward price of the dollar. Remember that domestic = Japanese interest rates (you are Japanese, after all), and foreign is U.S. interest rates. Go to www.bloomberg.com. Click on currencies. Click on International Bonds. Record the yield that exists for six-month U.S. treasuries and for 6-month Japanese governments. Use these interest rates in aaFXfwd to compute the fair forward price of a dollar.

5.24 You are a U.S. citizen. The spot exchange rate with Swiss Francs is $0.6495/SFR. The futures price of 1 SFR for delivery six months hence is $0.6624/SFR. The interest rate in Switzerland is 3%. Use aaFXfwd_repo_d to find the repo rate for the futures contract. How does the domestic repo rate you computed compare with the interest rate that exists in the United States for six-month debt instruments?

5.25 Suppose that the spot interest rate for four-month debt instruments is 5%, and the spot interest rate for one-year debt instruments is 6%. First find the unannualized interest rate for a four-month holding period. Use the Financial-CAD function aaFRAi to compute the implied forward rate. Note that the numbers to be entered into the discount factor cells are $1/(1+h)$, where h is the unannualized interest rate.

APPENDIX A Selling Short

It is simple to conceive of a long position in an asset, whether it be a physical or a financial asset. An investor first buys the asset. While he owns it, he has a long, or bullish position. He will sell it at a later date, hopefully at a higher price. Investors always like to buy low and sell high.

A short seller also likes to buy low and sell high. However, short sellers sell the asset first and later buy it back. They hope the price will decline during the time that they are short the asset. They are called "bears."

How can a short seller sell something he doesn't own? His broker borrows the asset from someone else. Thus, suppose A owns 100 shares of XYZ Corp., B wishes to sell 100 shares short, and individual C will buy these 100 shares. B's broker will borrow the shares from A and lend them to B, who then sells these shares to C, who in turn pays investor B for these shares. B hopes that at a subsequent date, XYZ will sell at a lower price. At that time B can buy the shares (at a price lower than the price at which they were sold short). B's broker takes the shares and returns them to A's account. All this is frequently done without A's knowledge, though when A opened his account, he must have consented to allow his shares to be borrowed in this manner.

What if A decided to take possession of the asset, or to sell it? The broker would then have to find another investor who owns the shares of XYZ, and borrow them from that person. It is possible, though unlikely, that no shares of a stock would be available for selling short. It is also possible that B would sell short, and some time after, *all* investors would want their shares; then the loan would be called, forcing B to buy the shares to close his short position. For example, suppose that B sells short and afterward there is a tender offer for the shares of XYZ Corp. In a tender offer, an individual or corporation makes an offer to acquire the existing shares of a company. If enough shares are tendered (including the shares of A, the original owner of the shares that B borrowed), there may be no shares available to be borrowed for short selling, and B will be forced to somehow buy the shares back to "cover" the short position. Another event that would force the short to cover is a proxy fight. Short selling creates more shares owned than the firm originally issued. In our simple example, both A and C own 100 shares. If (a contrived case) the firm has issued only the 100 shares that investor A bought, how can the shares be voted? Both investors A and C own shares, hence own voting rights.

For large corporations and normal voting events, there will be sufficient proxies to satisfy all long positions because many investors do not vote. The brokerage firm will find enough proxies to permit all to vote. But if there is an important vote, and there are no proxies available, the short seller will be squeezed (i.e., forced to buy the stock).

The broker of short seller B will not worry about investor B's ability to buy back the shares she sold short. That is because the proceeds of the short sale must be kept in investor B's account as collateral for the shares he has borrowed.

Moreover, not only does investor B not get to use the proceeds of the short sale, he also must meet margin requirements. Recently this was 50% of the value of the stock. For example, if 100 shares are sold short at $40/share, the short seller must keep the $4000 proceeds in the account as collateral and also deposit another $2000 to meet margin requirements (the $2000 can be in the form of securities, e.g., $2000 worth of another company's stock).

If the stock price starts to rise (to the short seller's dismay), the broker will demand that the short seller deposit more cash or securities.

Dividends offer a complication similar to voting rights. If the firm pays a dividend, who gets it: investor A or investor C? Both get it. The firm pays the dividend to investor C, and also, all short sellers must also pay dividends to the owners of the shares they have borrowed.

Short sales of common stock are subject to an "uptick rule." In the 1920s and earlier, ruthless speculators often sold short stock at prices below the last trade, driving quoted prices down. Shortly thereafter, they would disclose false negative information about the stock, which would cause others to sell their shares, perhaps even creating a selling panic. Then they would close their short positions at a profit.

To eliminate this type of abuse, and also to minimize the price-depressing effects of short selling, the Securities and Exchange Commission outlawed short sales of stock at prices below the last previous trade. An investor can sell short at the same price as the last trade only if the last previous change was an uptick. It is always permissible to sell short a price above the last trade. Thus if the last trade was at $40, you can sell short at $40.10, regardless of the trend in stock prices (assuming that there is a buyer at $40.10). You would be able to short sell at $40 only if the last prior trade was at a price less than $40.

Individuals sell short for many reasons. Bearish investors will sell short, though it is probably wiser to buy a put option. Losses are limited with a put; they are theoretically unlimited with a short sale.[1] Other investors sell short for tax reasons, as part of hedging strategies or for arbitrage purposes.

Note that in many of the valuation models of this book, we will assume that the short seller has full use of the proceeds of a short sale. In some cases, we can relax this assumption, and we will demonstrate the effect of doing so. Also, some large traders are able to negotiate with their brokers, hence can indeed make full use of the proceeds from selling short.

Teweles, Bradley, and Teweles (1992, pp. 162–180) contains a great deal of useful material on short selling.

Reference

Teweles, Richard J., Edward S. Bradley and Ted M. Teweles. 1992. *The Stock Market*, 6th ed. New York: John Wiley & Sons.

Note

[1]Consider an investor who sold short 100 shares of Diana Corp. on May 6, 1996, at a price of $46 per share. After all, the company had reported a loss of $0.18/share in its fiscal year ending March 31, 1995, and a loss of $0.80/share in its fiscal year ending March 31, 1996. But during this time, its stock price actually rose by 700%, from $5 3/4 on August 9, 1995, to $14 1/4 on February 21, 1996, to $46 on May 6, 1996. Obviously, some value-oriented investors might have considered it to be overvalued. Well, less than three weeks later, on May 24, 1996, the stock proceeded to rise to an intraday high of $120/share! Our hypothetical investor who sold short at $46 lost $74/share, or $7400 on the 100 original shares, if he covered his short position at $120! As a postscript, our investor's bearishness on Diana was not totally unwarranted: it plummeted to $39 1/4 on June 26, 1996, and $20 5/8 on August 19, 1996. By June 1997, it had been delisted from the NYSE and was selling for $3.125/share.

APPENDIX B Day Count Methods

There are several day count methods that are used to compute interest payments over different intervals of time. Thus, if an annual interest rate is quoted, the issue is how to compute interest over holding periods of less than a year.

- **30/360**: Generally assumes that there are 12 months of 30 days each in a year. If a month has 31 days, no interest is earned on the 31st day. Since there are 28 days in February, two extra days of interest are earned on the 28th day; in leap years, one extra day of interest is earned on February 29. One year equals 360 days. In this convention, three months is always one-quarter of a year; six months is always a half-a-year. For example, the period of time between January 28 and July 28 is half a year. Exceptions occur when the last day of the period is on the 31st of the month and the first day of the period is *not* the 30th or 31st. That is, the number of days between January 28 and March 31 is 63 (2 days in January, 30 days in February, and 31 days in March), and there is 0.175 of a year between those two dates (63/360). Oddly, there are also 63 days between January 28 and April 1 (2 days in January, 30 days in both February and March, and 1 day in April). Corporate bonds, municipal bonds, and agency securities generally use the 30/360 day count method when accruing interest.
- **Actual/360**: Count the actual number of days between two dates and divide by 360 to find the fraction of the year. In the United States this is called the "money market basis." Note that there is 1.0138889 year between two dates one year apart in this method (365/360); an annual money market rate of 4% on $100 would produce $4.05556 in interest.
- **Actual/365 (fixed)**: Count the actual number of days between two dates and divide by 365 to find the fraction of the year.

- **Actual/365 (actual) = U.S. government actual/actual = actual/actual**: Count the actual number of days between two dates and divide by 365 to find the fraction of the year in non-leap years. In a leap year, divide by 366. U.S government bonds accrue interest using the actual/actual method.
- **Euro 30E/360**: Always assumes that every month has 30 days and that there are 360 days in a year. No exceptions are made when the last day of the period is on the 31st day of the month. Thus, there are 62 days between January 28 and March 31 (2 days in January, and 30 in both February and March).

APPENDIX C Computing the Future Value of Carry Return Cash Flows

The theoretical forward price was derived in this chapter to be

$$F = S + CC - CR$$

If today is time 0, a carry return cash flow will be paid at time $t1$, and the delivery date is at time T, then this model may be expressed as follows:

$$F(0, T) = S[1 + h(0, T)] - \text{div}[1 + fh(t1, T)]$$

where

$F(0, T)$ = time 0 theoretical forward price for delivery at time T

S = spot price of the underlying asset

$h(0, T)$ = unannualized interest rate for the period from today until time T

div = cash flow that is received by the owner of the underlying asset

$fh(t1, T)$ = unannualized interest rate that will exist at time $t1$; the cash flow carry return is received at time $t1$, and it will earn this rate of interest until time T

The careful reader might ask how we can derive the theoretical forward price when we don't really know what $fh(t1, T)$ will be. In other words, at time 0, we don't know what amount of interest the cash flow carry return (the dividend) will be able to earn.

The answer comes from Equation (5.11):

$$1 + h(0, T) = [1 + h(0, t1)][1 + fh(t1, T)] \tag{5.11}$$

Solve Equation (5.11) for $1 + fh(t1, T)$:

$$1 + fh(t1, T) = \frac{1 + h(0, T)}{1 + h(0, t1)}$$

Thus, the theoretical forward pricing equation becomes

$$F = S[1 + h(0,T)] - \text{div}\left[\frac{1 + h(0,T)}{1 + h(0,t1)}\right] = [1 + h(0,T)]\left[S - \frac{\text{div}}{1 + h(0,t1)}\right] = FV[S - PV(\text{div})]$$

where FV is the future value operator and PV is the present value operator; $PV(\text{div})$ is the present value of the carry return cash flow.

This equation is operational at time 0 (today) because both $h(0,t1)$ and $h(0,T)$ are spot interest rates.

CHAPTER 6

Introduction to Futures

In this chapter, we present the essential features of another important derivative security, the futures contract. A considerable amount of terminology and institutional detail surround futures contracts. However, struggling with these details will pay off because these details provide the foundation to understand how futures contracts can be used to manage price risk.

At first, many people think that futures contracts are exactly the same as a forward contract. Although the two instruments are similar, they are not exact substitutes. There are two very important distinctions between these two types of contract. Recall that a disadvantage of a forward contract is that the firm faces the risk of default on the contract by the counterparty. An advantage of a futures contract is that the risk that counterparty default risk is essentially eliminated. However, this benefit comes at a cost. Unlike forward contracts, futures contracts are standardized, which means that they are not as flexible as forward contracts. These distinctions mean that both forward and futures contracts can provide unique risk-shifting benefits.

We begin this chapter with a detailed discussion of the features that distinguish futures contracts from forward contracts. Therefore, a good way to prepare for this chapter is to review Chapter 3, which provides an introduction to forward contracts.

6.1 Futures Contracts and Forward Contracts

Speculators use futures to a greater extent than forwards. Speculators buy futures contracts when they believe that the price of the good will rise. Speculators sell futures contracts when they believe that prices will fall. Hedgers also use futures contracts to manage their risk exposure, and arbitrageurs exploit situations when futures prices are sufficiently different from their theoretical values.

For both futures and forward contracts, one party agrees to buy something in the future from a second party; the second party agrees to sell it. The buyer of a contract, who is said to be long the contract, has agreed to buy (take delivery of) the good. The seller is said to be short the contract, and this person has the obligation to sell (deliver) the good at some time in the future. The contract specifies both the quantity and quality of the good, the price, the delivery date or dates, and the delivery location or locations.[1]

Futures and forwards are in zero net supply; for every buyer of a contract, there is a seller. The profits and losses realized in forward and futures contracting represent a zero-sum game; for every dollar one party makes, another party must lose a dollar (ignoring commissions). However, there are several ways in which futures contracts differ from forward contracts.

1. Futures contracts are standardized; only the price is negotiated. All December 2001 gold futures contracts are identical in that the amount of gold (called the **contract size**), quality of gold, delivery date, and place of delivery are specified. In contrast, all elements of forward contracts are negotiated; they are custom-made contracts that can be designed to meet the specific needs of the parties.

2. Futures contracts are usually more liquid than forward contracts, partly because they are standardized. Futures trade on futures exchanges. Refer to Table 1.2 for a list of futures exchanges in the United States. Table 1.4 presents a list of leading international futures exchanges. The party who is long a futures contract can always terminate that obligation by subsequently (but prior to the delivery date) selling a contract for the same good with the same delivery date. Similarly, a short can close that position by later buying the same futures contracts. Each of these actions is called **offsetting** a trade. For example, if you are long a December 2001 gold futures contract, you can close your position by selling a December 2001 gold futures contract. In contrast, with the exception of the FRA market and the forward foreign exchange market, most other forward markets are illiquid.

3. Because forward contracts are agreements between two parties, each party faces the risk that the counterparty will subsequently default. Futures traders need not worry as much about default risk. Once a futures price has been agreed upon and a trade completed, the exchange's clearinghouse becomes the opposite party to both the buyer and the seller.[2] In other words, when party A goes long a futures contract, he buys it from the clearinghouse; when party B goes short a futures contract, she sells it to the clearinghouse. The clearinghouse is always neutral; it is both long and short an identical number of contracts. The number of outstanding contracts is called **open interest**. Because the clearinghouse is a party to every trade, buyers and sellers of futures contracts need be concerned only about the financial integrity of the exchange on which the futures contract trades, *provided their futures commission merchant (FCM) does not fail.*[3] The clearinghouse guarantees all payments of profits as long as its member FCMs are solvent. There have been no futures exchange failures in the United States. Indeed, if a futures exchange ever becomes financially distressed, it has the power to directly assess its members to cover any shortfalls, and no U.S. futures exchange has ever had to resort to that move. The requirement that traders post margin and the process of marking to market (both discussed in the next section) protects the exchange by ensuring that the buyers and sellers abide by their contractual obligations.[4]

4. Most futures positions are eventually offset. Only a small percentage, perhaps 1–5%, of a futures contract's open interest ever results in delivery of the good. Indeed, some futures contracts, such as the Chicago Mercantile Exchange's Eurodollar futures contract and stock index futures, are **cash settled**.[5] When this is the case, there are no provisions for delivery of the good; only cash profits and losses are exchanged. In contrast, most forward contracts terminate with delivery of the specified good.

5. Perhaps the most important differences, at least for pricing purposes, are the issues of security deposits (margin) and the timing of cash flows (daily resettlement, or marking to market). No money changes hands initially in a forward contract. Two parties agree on a fair forward price for later delivery, and a cash flow occurs, or a profit or loss is realized, only on the delivery date.[6] At that time, the cash flow (profit) is defined as the difference between the forward price initially agreed upon and the actual spot price of the good on the delivery date.

While profits or losses on forward contracts are realized only on the delivery day, the change in the value of a futures contract results in a cash flow every day. This means that there is less default risk with futures contracts. A forward contract can build up a great deal of value prior to delivery. If the contract becomes a large liability for one of the parties, then that party might seek legal ways (or even illegal ways) to avoid meeting the contract's obligations. In contrast, the daily change in the value of a futures contract must be exchanged, so that if one party (the losing party) defaults, the maximum loss that will be realized is just one day's value change. Thus the incentive to default is greater for forwards than for futures.

Margin requirements and marking to market determine the cash flow consequences of futures contracts. We now discuss these important concepts in turn.

6.2 Margin Requirements for Futures Contracts

When two parties trade a futures contract, the futures exchange requires some good faith money from both, to act as a guarantee that each will abide by the terms of the contract. This money is called **margin**.[7] Each futures exchange is responsible for setting the minimum **initial margin** requirements for all futures contracts that trade there. The initial margin is the amount a trader must initially deposit into his **trading account** (also called a **margin account**) when establishing a position. Many futures exchanges use computer algorithms, the most popular of which is called SPAN (standard **p**ortfolio **an**alysis of risk), to establish initial margin requirements.[8] The algorithms analyze historical price data to derive what is believed to be a worst-case one-day price movement. An exchange can change the required margin anytime. If price volatility increases, or if the price of the underlying commodity rises substantially, the initial margin required will likely be increased.[9]

It is important to realize that the margin required for trading futures differs from the concept of margin when one is buying common stock or bonds. In the purchase of common stock or bonds, margin is the fraction of the asset's cost that must be financed by the purchaser's own funds. The remainder is borrowed from the purchaser's stock broker. Futures margin, however, is good faith money, or collateral, designed to ensure that the futures trader can pay any losses that may be incurred. Futures margin is not a partial payment for a purchase.

Beyond the initial margin, if the equity in the account falls to, or below, the **maintenance margin** level, additional funds must be deposited to bring the account back up to the initial margin level. Thus, if the initial margin required to trade one gold futures contract is \$1000, and the maintenance margin level is \$750, then an adverse change of \$2.60/oz. will result in a margin call. Because one gold futures contract covers 100 oz. of gold, a decline of \$2.60/oz. in the futures price will deplete the equity of a long position by \$260. The trader with losses must then deposit sufficient funds to bring the equity in the account back to the initial margin of \$1000. The margin that is deposited to meet margin calls is called **variation margin**. A trader who does not promptly meet a margin call will find his position liquidated by the FCM.

Note that once a trader has received a margin call, he must meet that call, even if the price has subsequently moved in his favor. For example, suppose the futures price of gold declines \$4/oz. on day t. On day $t+1$, the trader who is long a gold futures contract receives a margin call, regardless of the futures price of gold on day $t+1$. Even if the futures price has risen considerably (by more than \$1.50/oz.) since day t's close, the trader must still deposit \$400 in variation margin into his account.

FCMs will often set initial and maintenance margin requirements at levels higher than the minimum requirements specified by the exchanges. Margin requirements also differ for different traders, depending on whether the position is part of a spread, a hedge, or a speculative trade. Margin requirements on spreads and hedges are less than those on speculative positions. In a spread, a trader who is long one contract will also be short another related contract. The two contracts may be on the same good, but for different delivery months (called a **calendar spread**, or an **intermonth spread**), or they may be on two similar goods for delivery in the same month (called an **intercommodity spread**, or an **intermarket spread**). In a hedge, the futures trader either owns the good and is short a futures contract or is short the good and long a futures contract. Hedge traders must sign a hedge account agreement declaring that the trades in the account will in fact be hedges as defined by the Commodity Exchange Act of 1936 and as specified by the Commodity Futures Trading Commission (CFTC). Finally, FCMs sometimes set different margin requirements for contracts with the most nearby delivery date and for contracts with delivery in more distant months.

Some sample margin requirements for speculative positions, as required by one discount FCM in July 2000, are shown in Table 6.1. Note that they are always subject to change by either the broker or the exchange. There are different margin requirements for hedgers and for traders who have spread positions (either intracommodity calendar spreads or intermarket spreads). The exchange website on which a futures contract trades will provide its most recent minimum margin requirement.

Initial margin requirements do not have to be met with cash. Instead, Treasury bills can be used to satisfy original margin. Some exchanges and FCMs will also allow a bank letter of credit to meet initial margin requirements. Such a letter from the futures trader's bank guarantees the FCM the trader has sufficient funds to trade and also guarantees that the bank will make up any shortfall, generally by a wire transfer.

TABLE 6.1 Margin Requirements for Selected Futures Contracts: Speculative Positions, July 2000

Commodity	Exchange[1]	Initial Margin	Maintenance Margin
S&P 500	CME	$23,438	$18,750
Nikkei 225	CME	$6,750	$5,000
Japanese yen	CME	$4,212	$3,120
Deutsche mark	CME	$1,249	$925
Mexican peso	CME	$2,500	$2,000
Eurodollar	CME	$675	$500
T-bill	CME	$237	$175
T-bond	CBOT	$2,025	$1,500
10 year T-note	CBOT	$1,350	$1,000
Gold	CMX	$1,350	$1,000
Crude oil	NYM	$3,375	$2,500

Intermarket Spreads	Initial	Maintenance
Deutsche mark vs British pound	$1,485	$1,100
T-bill vs eurodollar	$237	$175
T-bond vs 10-year T-note	$1,080	$800
Crude oil vs heating oil	$1,418	$1,050
S&P 500 vs Value Line (2:5)	$24,250	$19,900

[1]CBOT, Chicago Board of Trade; CME; Chicago Mercantile Exchange; CMX; Commodity Exchange, Inc. (a division of the NYMEX); NYMEX, New York Mercantile Exchange.

Treasury bills and bank letters of credit are the preferred means of meeting initial margin requirements. Cash in a customer's cash account may or may not earn interest (check whether your FCM pays interest on your cash balance in your cash account!). But cash used to meet margin requirements *never* earns interest. Treasury bills and letters of credit allow the trader to earn interest on initial margin, and therefore, they are the preferred means of meeting the initial margin requirement. When securities such as Treasury bills are used to meet the initial margin requirement, all variation margin payments must be made in cash.[10]

6.3 MARKING TO MARKET

All futures traders' positions are **marked to market** daily. The process is also sometimes called **daily resettlement**. What it means is that every day, profits are added to, or losses are deducted from, the equity of a trader's account. In effect, a trader is offsetting his position every day, and realizing each day's profit or loss. The profits and losses are based on changes in the **settlement price**, or closing futures price, for the contract of interest. All profits that increase the margin account balance above the initial margin amount can be withdrawn daily and spent by the trader. Losses deplete the equity in the account, until there is a margin call, at which time variation margin must be deposited to bring the account balance back to the initial margin level.

The process of daily resettlement makes a futures contract equivalent to a time series of one-day forward contracts. Recall that the profit or loss per unit of the underlying asset for a forward contract is $|F(0,T)-S(T)|$, where $F(0,T)$ is the futures price when the contract is originated and $S(T)$ $[=F(T,T)]$ is the spot price on the delivery date. Well, the daily mark-to-market cash flow for a futures contract is $|F(0,T)-F(1,T)|$: that is, the change in the futures price from one day to the next. But for a forward contract calling for delivery one day hence, it is the case that $F(1,T)=S(T)=F(1,1)=S(1)$. Thus, the cash flows arising from a futures contract are the same as if a trader had a forward contract calling for delivery one day hence; every day the forward contract is settled up, profits and losses are realized, and a new one-day forward contract is created. The final sum of the cash flows for the futures contract equals the one-time profit or loss for the forward contract.

Marking to market greatly reduces counterparty default risk. The most a trader has at risk is her one-day profit. There is no buildup of asset value (for one party) and liability value (for the other) as with forward contracts.

The entire daily resettlement process is illustrated with the following example. On November 6, 2001, you sell one gold futures contract for delivery in December 2001. You sell the contract at 10 A.M., when the futures price is \$285/oz. The initial margin requirement is \$1000, and that sum of money is transferred from your cash account to your margin account. The settlement price at the close on November 6 is \$286.40/oz. Your account is marked to market, and your equity at the close is \$860. The futures price rose by \$1.40/oz, and one contract covers 100 oz of gold; therefore, you have lost \$140 on the short position.

On all subsequent days, the account is marked to market. If the futures price falls, your equity rises. If the futures price rises, your equity declines. Maintenance margin calls will have to be met if your account equity falls to a level equal to or below \$750. Table 6.2 illustrates the cash flow consequences of marking to market for this example.

On November 7, a further mark-to-market loss (beyond the \$140 lost on the day you opened your position) of \$240 is realized. This brings your equity in the account down to only \$620. A maintenance margin call occurs, and you must deposit \$380 to bring your equity in the account

TABLE 6.2 Example of the Mark-to-Market Process[1]

Date	Settlement Price	Initial Cash Balance	Mark-to-Market Cash Flow	Equity	Maintenance Margin Call	Final Cash Balance	Final Equity
11/6	$286.4	$1000	$–140	$860	—	$1000	$860
11/7	$288.8	$1000	$–240	$620	380	$1380	$1000
11/10	$289.0	$1380	$–20	$980	—	$1380	$980
11/11	$288.6	$1380	$40	$1020	—	$1380	$1020
11/12	$290.7	$1380	$–210	$810	—	$1380	$810
11/13	$292.8	$1380	$–210	$600	400	$1780	$1000
11/14	$292.8	$1780	$0	$1000	—	$1780	$1000
11/17	$292.7	$1780	$10	$1010	—	$1780	$1010
11/18	$295.8	$1780	$–310	$700	300	$2080	$1000
11/19	$296.1	$2080	$–30	$970	—	$2080	$970
11/20	$297.1	$2080	$–100	$870	—	$2080	$870
11/21	$296.4	$2080	$70	$940			

[1]*The situation*: at 10 A.M. on November 6, 2001, you sell one December 01 gold futures contract at a futures price of $285/oz. The initial margin requirement is $1000 and the maintenance margin level is $750. It is assumed that all margin calls are met at the end of the trading day that the violation occurs.

back to the initial margin level of $1000. Note that in practice, you would likely receive the margin call just before the market opens on November 10. If you do not pay the $380 in cash, or close your position, your FCM will liquidate the position. Some FCMs will sometimes grant customers a few days to meet a margin call. The same FCMs, under different situations, will allow only a few hours. Note also that it does not matter what happens to the futures price on November 10; you still must meet the maintenance margin call in cash.

At the close of trading on November 11, your equity in the account has risen to $1020. The difference between your equity and the initial margin required is called **excess**. At the close of November 11, your excess is $20. Although some FCMs discourage the practice, you can withdraw excess if it is positive. Doing so is worthwhile because cash in your margin account does not earn interest for you (it will earn interest for your FCM though). By withdrawing the excess and transferring it back to your cash account, you can earn interest on the money.

The daily resettlement process continues, with margin calls at the close of trading on November 13 and again on November 18. Finally, on November 21, you offset your trade by buying one gold futures contract for December 2001 delivery at a futures price of $296.4/oz. The final equity (after the November 21 profit of $70 is accounted for) of $940 is then transferred back to your cash account.

There are at least three ways to compute your loss on this trade; all of them provide the same dollar loss amount. First, since you went short one contract at a futures price of $285/oz, you could offset the trade at a futures price of $296.40. This would mean a loss of $11.40/oz. Since the futures contract covers 100 oz. of gold, your loss on the trade would be $1140.00.

Second, you might add all the cash outflows between your cash account and your margin account. You originally transferred the initial margin of $1000. You also had three margin calls of $380, $400, and $300. Adding, the total amount flowing out is $2080 (which equals the final cash balance in your margin account at the time you offset your trade). If you subsequently transferred your final equity of $940 from your margin account to your cash account, your total loss would be $1140 ($2080 − $940).

Finally, you could just add up the daily mark-to-market cash flow amounts into and out of your margin account. The sum of these cash flows totals −$1140.

It is important to realize that the timing of the cash flows for the futures trade depends on how the initial margin requirement is satisfied and that these cash flow streams differ from those of a forward contract. Figure 6.1a illustrates the timing of these different cash flow streams. For instance, had you sold a forward contract on 100 oz. of gold at a forward price of $285/oz., there would be

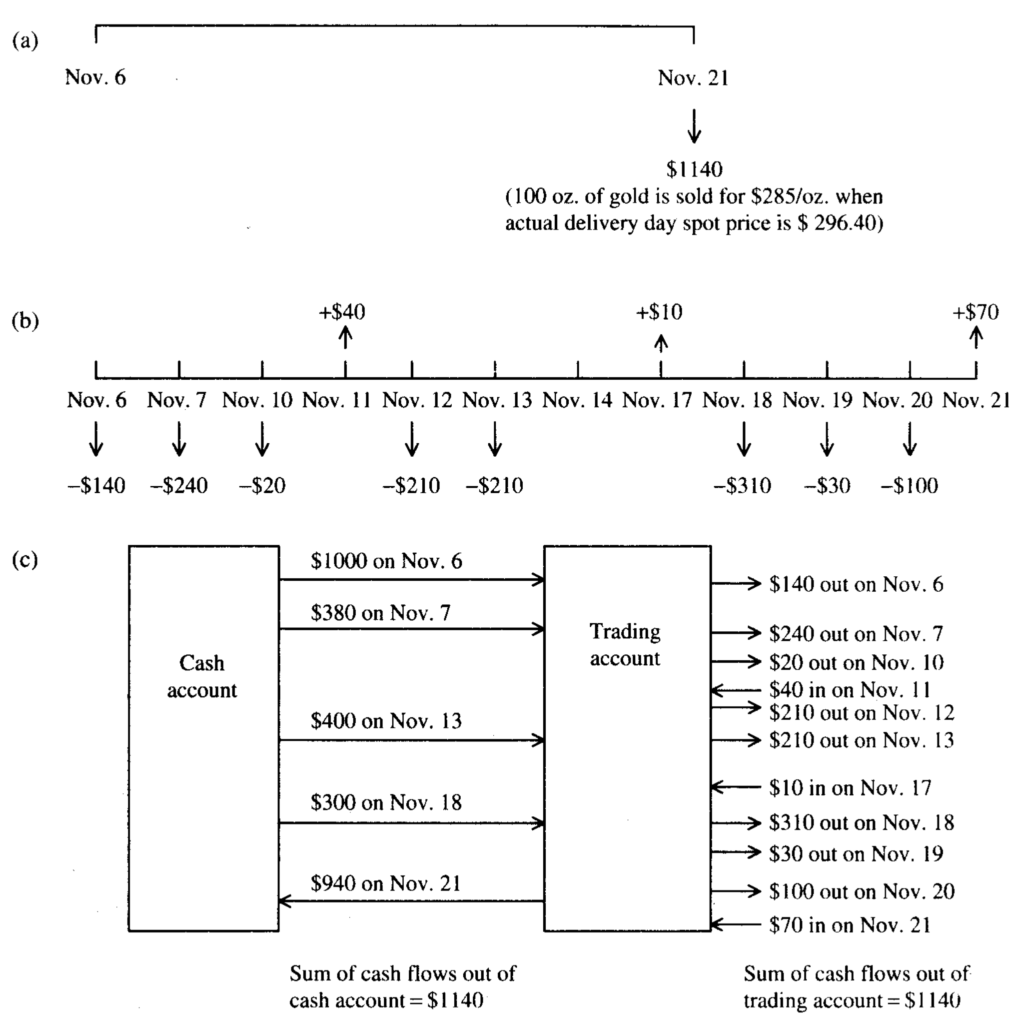

Figure 6.1 The timing of cash flows resulting from (a) the sale of a forward contract, (b) the sale of a futures contract using T-bills or a bank letter of credit for initial margin, and (c) the sale of a futures contract using cash for initial margin (see also Table 6.2).

no cash flow until the agreed upon delivery date (assuming that you did not negotiate good faith money on the initial date of November 6). If the delivery date of the forward contract happened to be November 21, and the spot price of gold on that day happened to be \$296.40/oz., then the only cash flow we define is the cash outflow of \$1140 on November 21: $F(0,T)-S(T)=285.00-296.40=-\$11.40$/oz. The entire loss is realized on November 21, as Figure 6.1a illustrates.

For a futures contract, however, losses are realized gradually over time, depending on how the initial margin requirement was met. Figure 6.1b illustrates the cash flows that would have resulted if you had used a T-bill or a bank letter of credit to satisfy the initial margin requirement on the gold futures contract in the foregoing example. There is no initial cash flow at 10 A.M. on November 8. If both parties pay nothing for the futures contract, it must not be worth anything; in other words, it has a value of zero. Subsequently, the cash flows are the amount by which the value of the contract changes each day. Essentially, then, it is as if the contract is bought back each day at its settlement price and then immediately resold at each day's settlement price. With a T-bill or bank letter of credit in your margin account (earning interest), the mark-to-market cash flows represent variation margin. Thus, at the end of the day on which the contract was sold (November 6) you would have to pay \$140 in cash. On November 7 you must pay \$240 in cash. On each subsequent day cash will either be debited (losses, or cash outflows) or credited (profits, or cash inflows) to your account. The total cash outflow comes to \$1140. After it has been marked to market each day, a futures contract again has zero value; all changes in value are realized daily through the mark to market process.

Figure 6.1c illustrates how cash flows occur if \$1000 in cash is used to meet the initial margin requirement. Margin calls take place on November 7, 13, and 18. Either cash will be withdrawn from your cash account or your FCM will quickly notify you to deposit funds into your cash account so that the amount of cash in your trading account can get back to the initial margin requirement amount of \$1000.

Like a forward contract, a futures contract has no value on its initiation day. The contract is merely an agreement for one party to buy and the other party to sell something at a later date. On the contract initiation day, presumably both the contract buyer and seller believed that the futures price of \$285/oz. was fair. If neither party paid anything for the contract on November 6, it must have been a valueless contract.

But there is a difference between the value of a forward contract and a futures contract on all days subsequent to the day on which it was initiated. For a forward contract, there is no profit or loss on any day except on the delivery date (assuming that it is held until the delivery date). As the price for forward delivery changes, so will the value of a forward contract. There can be a large buildup in the value of a forward contract if the forward price changes considerably from the original forward price. The forward price stays fixed throughout the contract's life. If a newly created forward contract has a forward price that is different from the forward price that was originally negotiated, the original contract will have positive or negative value. The value of a long position will be the negative of the value of a short position.

In contrast, changes in futures prices are settled up daily. Any change in the value of the futures contract is realized by the parties through the daily resettlement process. There is no buildup in the value of a futures contract. The maximum change in value occurs over a one-day interval, just prior to the instant at which it is marked to market. Once a futures contract has been marked to market, its value reverts to zero.

Computing a rate of return on a futures trade is often a problem. On one level of analysis, we can say that the investor has no initial investment. The initial margin is not an investment; it is good faith collateral. The trader can post T-bills or a bank letter of credit to satisfy the requirement. In this case, there is no initial investment, and any rate of return on an initial investment of zero dollars is infinite.

If, instead, the relevant cash flows are the daily mark-to-market cash flows (−140, −240, −20, . . . , + 70) are used to compute the internal rate of return, then again there is no answer, since it is impossible to analyze the internal rate of return for a series of cash inflows and outflows with more than one sign change.[11]

A common solution for computing a holding period rate of return on a futures contract is to consider the initial outlay as the cash outflow required to meet the initial margin. The total profit or loss on the trade is divided by the initial margin cash outflow. This approach suffers from many drawbacks, the most serious of which is that it adds cash flows occurring on different dates (variation margin, or maintenance margin calls). In other words, there is no accounting for the timing of the daily resettlement cash flows. Still, if the initial margin of $1000 in our example was made with cash, we would conclude that the trade resulted in a −114% rate of return. Your initial cash outflow (initial investment) was $1000. If you lost all of that, we would say you lost 100% of your initial investment. But you lost *more than* your initial investment. Thus:

$$\text{rate of return} = \frac{\text{profit or loss}}{\text{initial investment}} = \frac{-1140}{1000} = -114\%$$

Note that this − 114% rate of return was achieved during a 15-day holding period. It is *not* annualized.

6.4 Basis and Convergence

Two important concepts in futures trading are basis and convergence.

Basis is usually defined as the spot price (cash price) minus the futures price.[12] There will be a different basis for each delivery month for each contract. In a "normal market," basis will be negative, since futures prices normally exceed spot prices. The reason for this is the theoretical cost-of-carry pricing model, which applies to futures as well as forwards. In an "inverted market," basis will be positive.

Basis will approach zero as the delivery date nears. At the close of trading on the delivery date, basis must equal zero.[13] If the spot price of the deliverable asset at the delivery location on the delivery date does not equal the futures price, there will be an arbitrage opportunity. If the basis was positive on the delivery day, an arbitrageur could buy the futures contract and sell the cash good that is deliverable into the contract.

EXAMPLE 6.1 Suppose that an instant before trading ceases on the delivery date, the spot price of deliverable grade gold at a delivery location is $290/oz. and the futures price of the expiring contract is $289/oz. (basis = + 1). Ignoring transactions costs, an arbitrageur could buy the gold futures contract. Then, to satisfy the terms of the contract, he would take delivery of the gold and pay $289/oz. He would then sell the gold in the spot market for $290/oz, realizing the $1/oz. arbitrage profit. A similar arbitrage trade could be done if the basis was negative just before the end of trading.

The process of basis moving toward zero is called **convergence**. Regardless of today's basis, because of the possibility of arbitrage, we know that at the close of trading on the delivery date, the basis will be zero for that contract.

Figure 6.2 illustrates the process of convergence for four commodities during 1996. Note how the basis always declines towards zero as delivery nears.

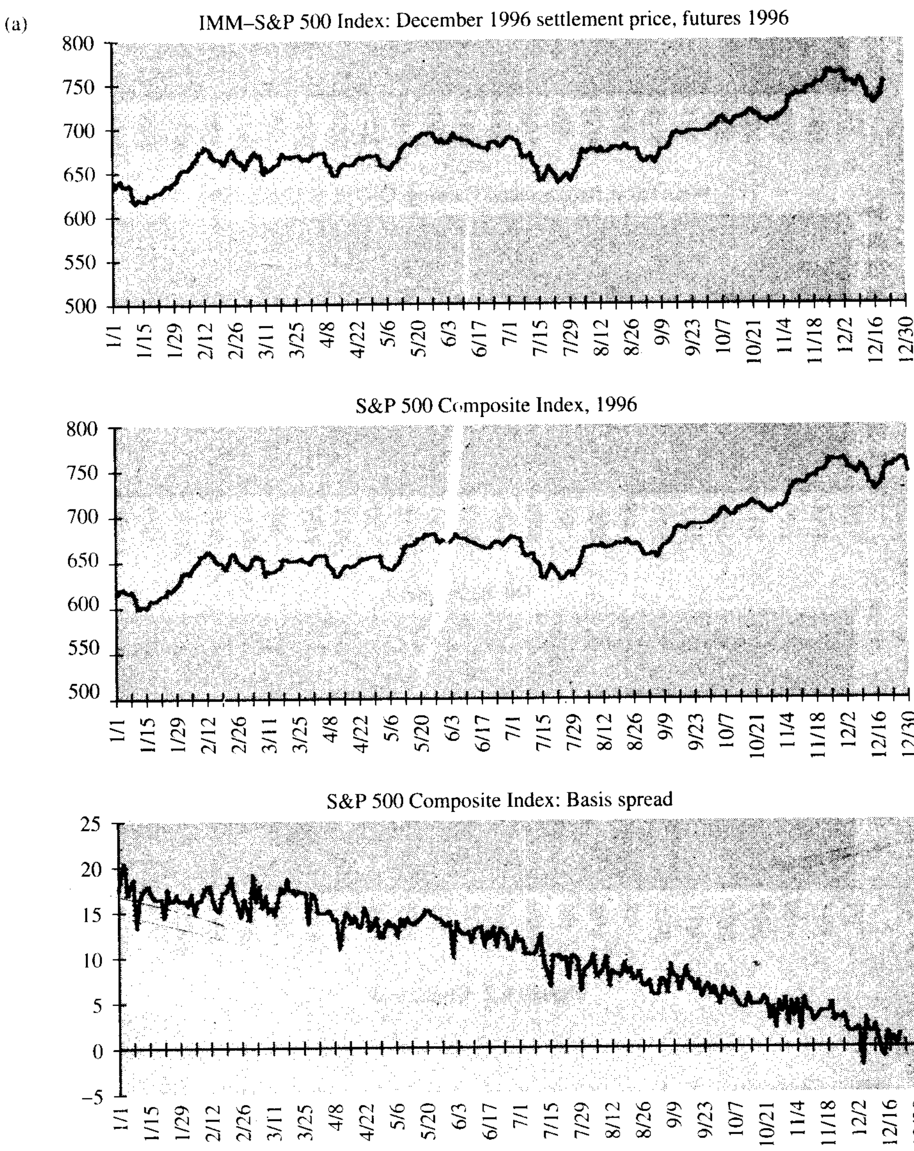

Figure 6.2 Convergence for four commodities during 1996: (a) S&P 500 Index, (b) oil, (c) yen, and (d) Eurodollars.

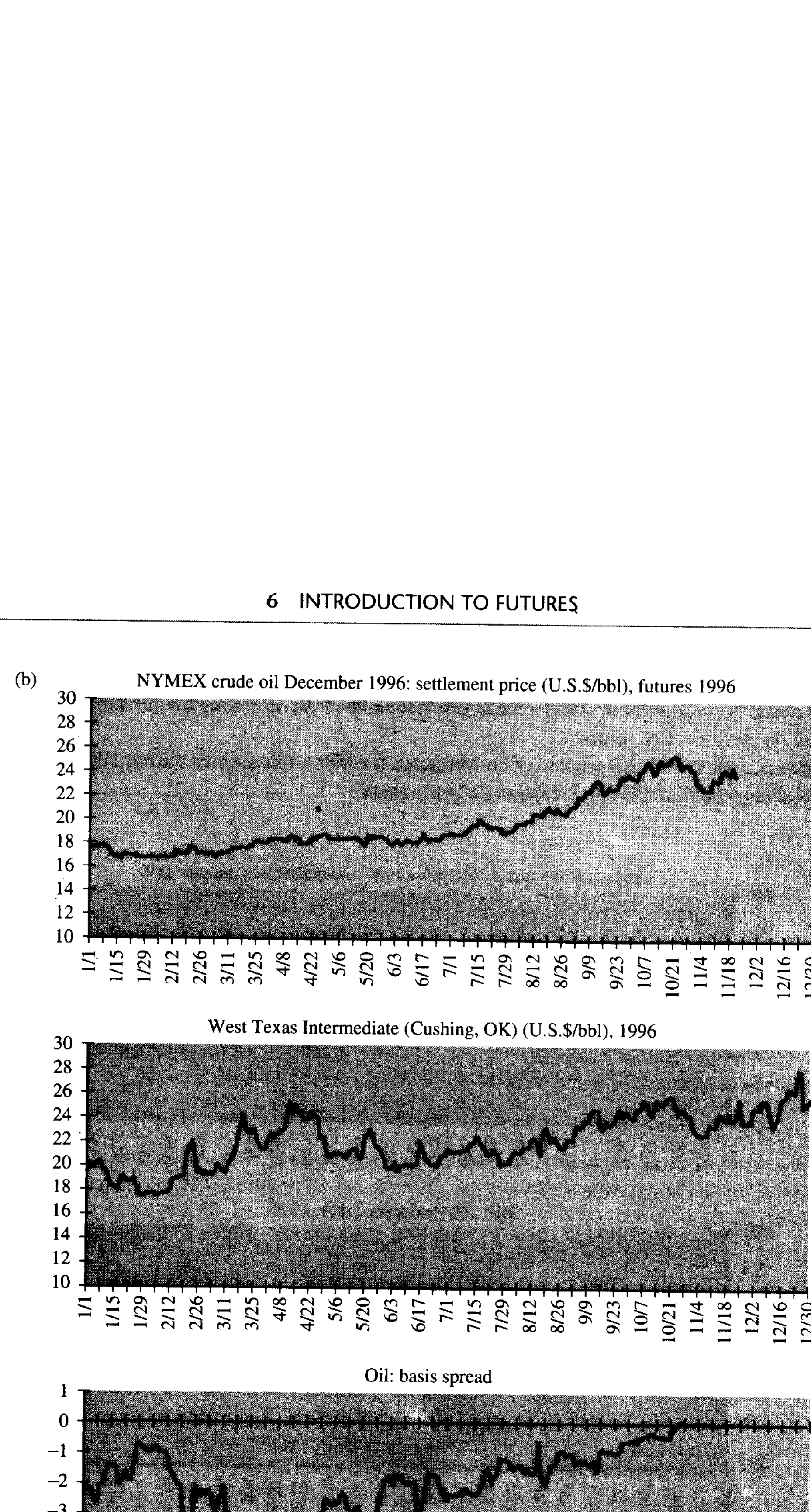

Figure 6.2 Continued.

Regardless of whether the basis is initially positive or negative, and regardless of how the spot price moves, convergence will occur. It is also possible that the basis will fluctuate between being positive and negative as time passes. Convergence will still take place. Figure 6.3 illustrates some different scenarios.

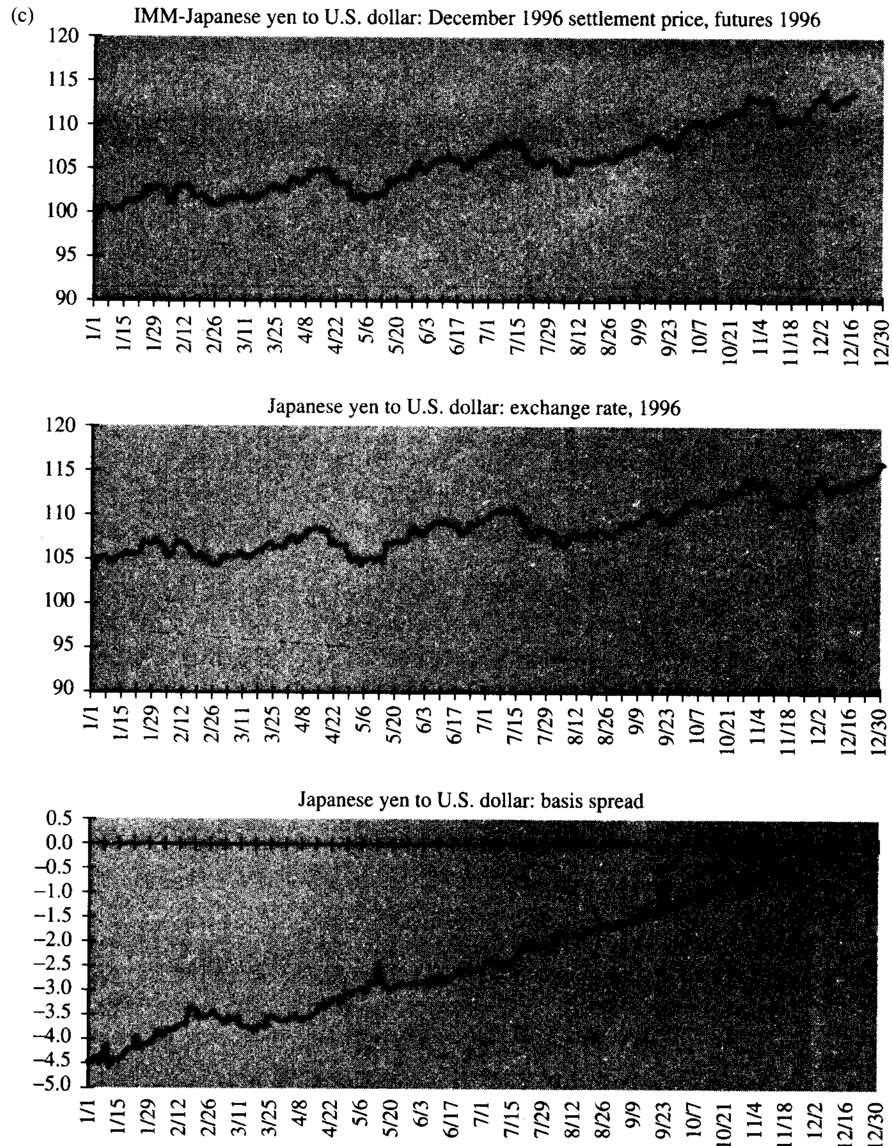

Figure 6.2 Continued.

6.5 Should Futures Prices Equal Forward Prices?

You now know about the differences that exist between forward contracts and futures contracts. In Chapter 5, you learned about how theoretical forward prices are computed. You might now wonder whether theoretical forward prices and futures prices are equal. In this section, we study this

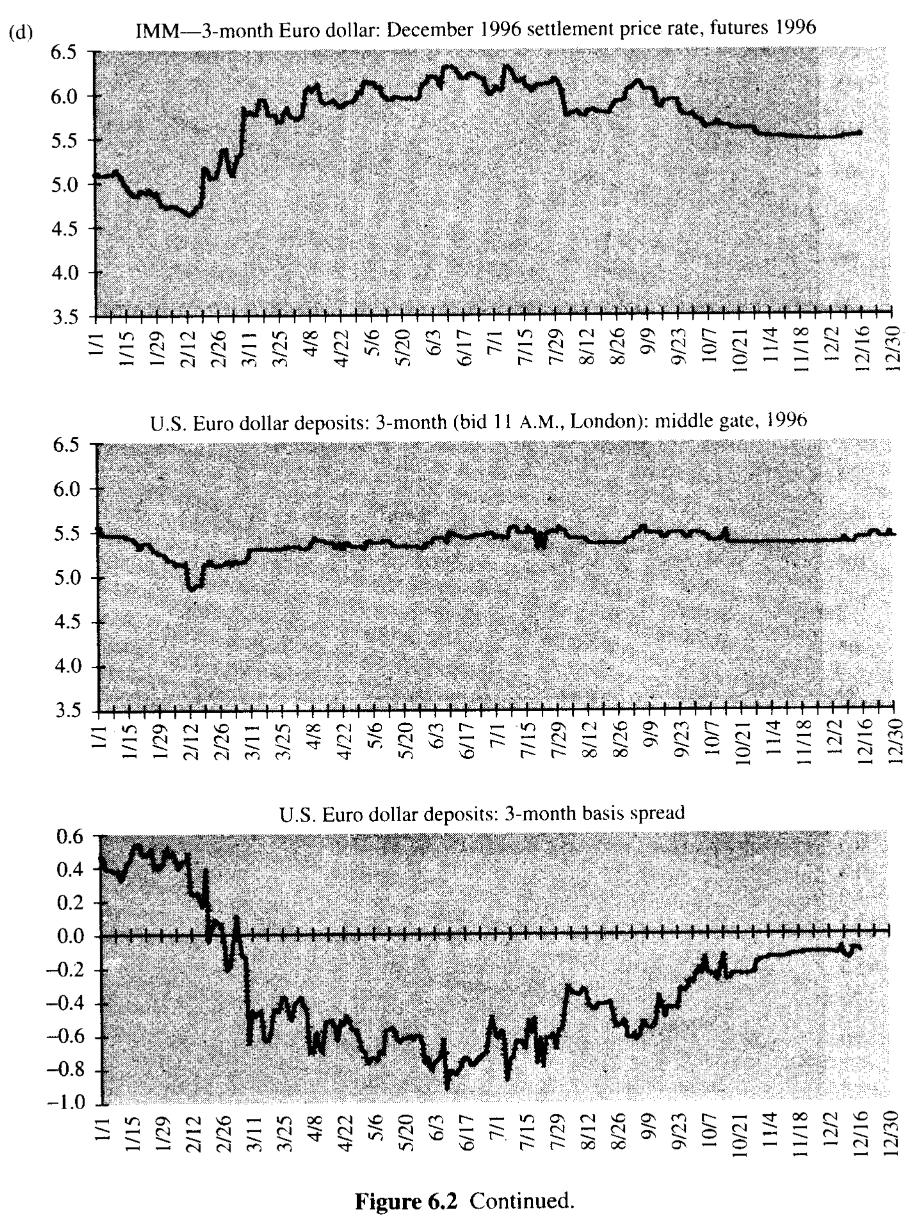

Figure 6.2 Continued.

question. An early study of futures and forward contracts is Black (1976). Cox, Ingersoll, and Ross (1981) is a rigorous further examination of the relationship between futures and forward prices. Many of their results, and other theoretical perspectives, are derived in Richard and Sundaresan (1981) and Jarrow and Oldfield (1981).

Basically, the results are obtained because if tomorrow's interest rate is known today, a futures contract can be transformed into a forward contract. If futures can be transformed into forwards,

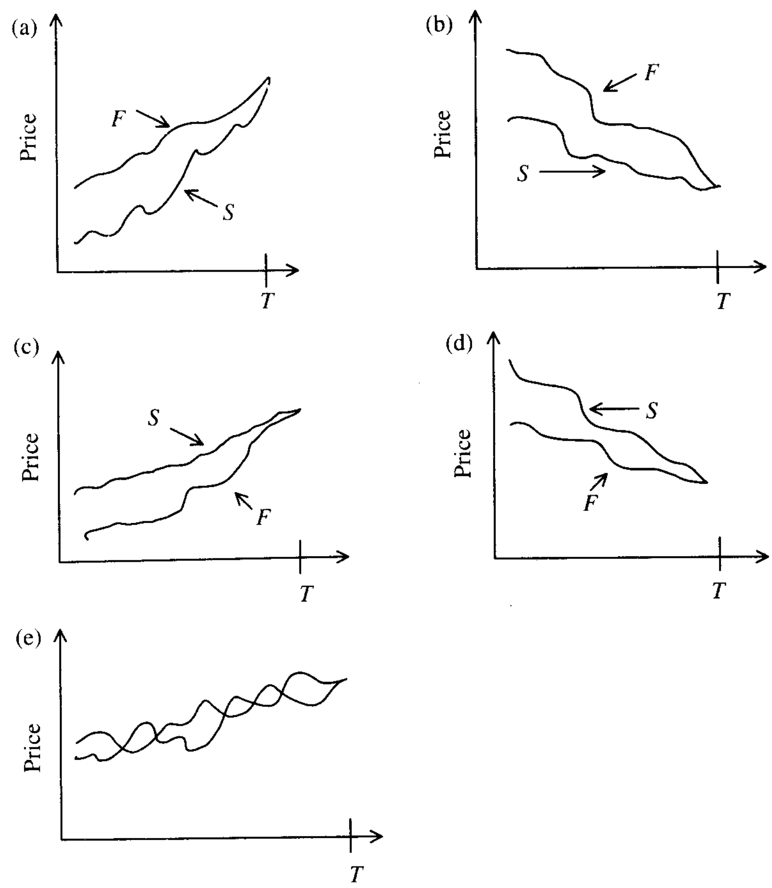

Figure 6.3 Convergence occurs regardless of the basis, and regardless of how the spot price changes over time. (a) Negative basis; spot price rises. (b) Negative basis; spot price declines. (c) Positive basis; spot price rises. (d) Positive basis; spot price declines. (e) Basis alternates between positive and negative.

then we can conclude that futures prices and forward prices will be identical. Note that we are ignoring default risk, transactions costs, taxes, and other market imperfections.

Proposition I. *In a perfect market with nonstochastic (known) interest rates, futures prices equal forward prices.*

We can prove this by example. Today is time 0. Define $r_{t1,\,t2}$ as the interest rate from time $t1$ to time $t2$. For example, assume that you know that at time 1, the three-period interest rate will be $r_{1,4}=10\%$/period. You also know that the two period interest rate at time 2 will be $r_{2,4}=9\%$/period. Finally, you know that the one-period interest rate at time 3 will be $r_{3,4}=8\%$/period. Assume that you satisfy the initial margin requirement to trade a futures contract with a bank letter of credit. Today's futures price for 100 oz. of gold is \$300/oz. You do not know what futures prices will be at subsequent times 1, 2, 3, and 4 (the delivery date). You buy $1/(1+r_{1,4})^3 = 0.7513148$ of a futures contract at the time 0 futures price of 400.[14]

Suppose that at time 1, the futures price rises to 310. The mark-to-market cash flow on one futures contract is \$1000. Your profit on 0.7513148 futures contract is \$751.3148. You invest this profit at the three-period rate of 10% per period. So at time 4, it will have grown to \$1000. You also buy enough additional futures contracts to be left with $1/(1+r_{2,4})^2=0.84168$ futures contract. In other words, buy 0.0903652 future contracts at the new futures price of 310.

At time 2, the futures price rises to 315. The mark-to-market cash flow on one futures contract is \$500. Your profit, however, is (0.84168×500=) \$420.84, because you are long 0.84168 contract. You invest this profit at the two-period rate of 9%/period. At time 4, this will have grown to \$500. You also buy enough additional futures to bring your total to $1/(1+r_{3,4})=0.9259259$ contract. In other words, buy 0.0842459 additional contract at the futures price of 315.

At time 3, the futures price falls to 290. The mark-to-market cash flow on one futures contract is –\$2500. You thus lose \$2314.8148 on your long position of 0.9259259 contract. You borrow this amount for one-period at the one-period rate of 8%. At time 4, your debt will equal \$2500. You also buy 0.0741741 additional futures contract so that you are now long one contract.

At time 4, owing to convergence, the final futures price equals the \$285 spot price of gold. Your mark-to-market cash outflow is \$500. Note that you had zero cash flows at times 1, 2, and 3 because all losses were borrowed and all mark-to-market gains were lent. Your cash flow at time 4 equals +\$1000+\$500−\$2500−\$500=−\$1500. But this total cash flow is exactly the same as that of a forward contract, which also has only one cash flow (the total profit or loss at delivery). In other words, had you bought a forward contract at time 0 at a forward price of \$300, with the delivery date spot price of gold closing at \$285, the value of the long forward contract on the delivery date would have been −\$1500.

Since the same analysis can be done with any *known* series of interest rates, and any realization of futures prices (which are stochastic as of time 0), we can conclude that in a perfect market with known interest rates, futures prices must equal forward prices at any date. If they were unequal, an arbitrageur could sell the contract (futures or forward) with the higher price and buy the contract with the lower price. When interest rates are known, a futures contract can be converted into a forward contract by trading in the manner just demonstrated. Basically, to convert a futures into a forward, the trader should always be long or short the present value of a futures contract.

Proposition II. *If interest rates are stochastic, and if corr(ΔF, Δr) > 0, futures prices will exceed forward prices.*[15] *If corr(ΔF, Δr) < 0, futures prices will be less than forward prices.*

If futures prices rise, long positions profit. If corr(ΔF, Δr) > 0, then at the same time interest rates rise, the mark-to-market cash inflows can be reinvested at the higher interest rates. It also follows that if changes in futures prices are positively correlated with interest rates, the losses that long positions experience when futures fall can be borrowed at lower interest rates. These two effects make long futures positions more valuable than long forward positions when corr(ΔF, Δr) > 0. Because of this, investors will bid up futures prices relative to forward prices when there is a positive correlation between futures price changes and interest rate changes.

If, on the other hand, corr(ΔF, Δr) < 0, then an increase in futures prices means that the mark to market profits of a long position will be reinvested at lower interest rates. Furthermore, if futures prices decline, they will tend to do so along with an increase in interest rates. The daily resettlement losses of a long futures position will then have to be borrowed at higher interest rates. Thus, if changes in futures prices are negatively correlated with changes in interest rates, a long futures position is less desirable than a long forward position. To induce investors to buy futures, the futures price will have to be less than the forward price.

EXAMPLE 6.2 This example utilizes the data from the example provided in connection with Proposition I, except if futures prices rise, interest rates and expected interest rates decline. The example demonstrates that because corr(ΔF, Δr) is negative, a long futures position is inferior to a long forward position if both the futures and forward prices are \$400/oz. at time 0.

At time 0, the futures price is \$400. Investors expect that at time 1, the three-period interest rate will be 10%; this is denoted as $E_0(r_{1,4})=0.10$. They also expect that at time 2, the two-period rate will be 9%; $E_0(r_{2,4})=0.09$. Finally, they expect that at time 3, the one-period rate will be 8%; $E_0(r_{3,4})=0.08$. You buy $1/[1+E_0(r_{1,4})]^3=1/(1.1)^3=0.7513148$ futures contract at $F_0=400$.

At time 1, the futures price rises to \$410. Your profit is (\$1000)(0.7513148) = \$751.3148. The spot three-period interest rate is 9%; $r_{1,4}=0.09$. (Note that the increase in the futures price is accompanied by a decline in interest rates.) You lend your \$751.3148 profit for three periods at 9%/period. Investors now expect that the two-period rate at time 2 will be 8%; $E_1(r_{2,4})=0.08$. You buy 0.106024 additional contract at $F_1=410$, putting yourself long $1/[1+E_1(r_{2,4})]^2=1/(1.08)^2=0.8573388$ futures contract. Investors also now expect that the one-period rate at time 3 will be 7%; $E_1(r_{3,4})=0.07$. Both the spot interest rate and all expected rates are thus lower than what investors had originally predicted.

At time 2, the futures price rises to \$415. Your profit is (\$500)(0.8573388) = \$428.6694. The spot two-period interest rate is 7%; $r_{2,4}=0.07$. You lend your \$428.6694 profit for two periods at 7%/period. Investors now expect that the one-period rate at time 3 will be 6%; $E_2(r_{3,4})=0.06$. You buy 0.0860574 additional contract at $F_2=415$, so that you are now long $1/[1+E_2(r_{3,4})]^1=1/1.06=0.9433962$ futures contract. The spot rate and expected rates are even lower than what investors had expected earlier.

At time 3, the futures price falls to \$390. Your loss is (\$2500)(0.9433962) = \$2358.4905. The spot one-period interest rate is 7%; $r_{3,4}=0.07$. You borrow the \$2358.4905 for one-period at 7%/period. You buy 0.0566038 additional contract at $F_3=390$, so that you are now long 1 futures contract. The spot rate is higher than what had been expected at time 2.

At time 4, the futures price falls to \$385. Your loss is \$500. Now repay the money you borrowed at time 3, and get repaid the loans you made at times 1 and 2. The total time 4 cash flow is thus $-\$500-\$2358.4905(1.07)+\$428.6694(1.07)^2+\$751.3148(1.09)^3=-\$1559.8268$.

Because corr(ΔF, Δr) < 0, this futures loss is bigger than it would have been if you had gone long a forward contract. The loss for a long forward position is −\$1500 and the loss for the long futures position is −\$1559.8268. Thus, the example illustrates that if investors believe that corr(ΔF, Δr) < 0, and the futures price equal the forward price, no one will go long futures. The futures price must be below the forward price if futures price changes and interest rate changes are negatively correlated. If the futures price was equal to the forward price in this example, arbitrageurs would sell futures and buy forwards, and realize an arbitrage profit in time 4. With futures prices and interest rates evolving as they did in the example, the arbitrageurs' profit is \$59.8268.

As usual, the foregoing results apply only in perfect markets. Transactions costs, taxes, liquidity, and contract indivisibilities all work to weaken the stated propositions. The cost-of-carry pricing model is a forward contract pricing model. When it is used to estimate futures prices, we must assume that daily resettlement of futures contracts has no effect on prices. Proposition I demonstrates that if future interest rates are known, then futures prices and forward prices are equal.

If markets are imperfect, but the correlation of futures price changes and changes in interest rates is zero, then futures prices and forward prices should be equal. Marking to market should have no effect on futures prices relative to forward prices. There are some commodities that have both futures and forward markets, and empirical research has concluded that their prices are so close that the effect of daily resettlement appears to be an insignificant institutional matter. We recommend that you always think that theoretically, futures prices and forward prices are equal.

6.6 FUTURES CONTRACTS, EXCHANGES, AND REGULATION

As of January 2001, futures and options contracts traded on 59 different futures exchanges in 29 different countries, including 12 in the United States and 7 in Japan. Each January, *Futures* magazine publishes a Source Book containing a summary overview of the contracts.

Table 6.3 gives the names, addresses, phone numbers, and websites of the major futures exchanges in the United States. Most, if not all, offer many publications describing the contracts

TABLE 6.3 Addresses of Major U.S. Futures Exchanges and Related Agencies and Organizations

1. Chicago Mercantile Exchange
   30 South Wacker Drive
   Chicago, IL 60606
   312-930-1000
   Website: www.cme.com/

The CME has available literally hundreds of publications, films, slides, videos, and so on. The Merc's Education Center offers inexpensive (about $150, on average) courses on futures, futures options, technical analysis, hedging, and even computer courses; you just have to get to Chicago—though many courses are taught in other cities, too.

The International Monetary Market (IMM) and Index and the International Option Market (IOM) are divisions of the CME.

2. Chicago Board of Trade
   141 West Jackson
   Chicago, IL 60604
   800-THE-CBOT (Publications Department)
   312-435-3500
   Website: www.cbot.com

The CBOT offers an immense amount of literature on futures. Request their publications catalog. The MidAmerica Commodity Exchange (MidAm) is an affiliate of the CBOT, and their phone number is 312-341-3000.

3. New York Board of Trade
   174 Hudson Street, 6th Floor
   New York, NY 10013
   212-625-6669
   Website: www.nybot.com

(Continued)

TABLE 6.3 Continued

The NYBOT is the parent company of the Coffee, Sugar & Cocoa Exchange, Inc. (CSCE) and the New York Cotton Exchange (NYCE). Through its two exchange and their subsidiaries including the New York Futures Exchange (NYFE), FINEX and Citrus Associates, the NYBOT offers a wide variety of agricultural (such as cotton and frozen orange juice) and financial (the U.S. dollar index and the NYSE composite index) products.

4. New York Mercantile Exchange (NYMEX)
   One North End Avenue
   World Financial Center
   New York, NY 10282
   212-299-2000
   Website: www.nymex.com

The NYMEX specializes in energy futures and options on energy futures. The COMEX (Commodity Exchange), which became a subsidiary of the NYMEX in 1994, specializes in precious metals futures and futures options.

5. Kansas City Board of Trade
   4800 Main, Suite 303
   Kansas City, Missouri 64112
   800-821-5228
   816-753-7500
   Website: www.kcbt.com

Futures on wheat, natural gas, and the Value Line Index trade on the KCBOT.

6. Commodity Futures Trading Commission (CFTC)
   Three Lafayette Centre
   1155 21st St. NW.
   Washington, D.C. 20581
   202-418-5498
   202-418-5430 (Division of Trading and Markets)
   202-418-5260 (Division of Economic Analysis)
   Website: www.cftc.gov

7. National Futures Association (NFA)
   200 West Madison St., Suite 1600
   Chicago, IL 60606
   800-621-3570
   312-781-1300
   312-781-1370
   Website: www.nfa.futures.org

8. Futures Industry Association
   2001 Pennsylvania Ave. NW., Suite 600
   Washington, D.C. 20006-1807
   202-466-5460
   Website: www.fiafii.org

The FIA, founded in 1955, is the major trade association for all organizations with an interest in futures markets. It offers publications and courses on futures trading. It also presents Futures and Options Expo every October, in Chicago. In 1988 the FIA established the Futures Industry Institute to provide educational information about the futures, options, and other derivatives markets; its phone number is 202-223-1528.

traded there, as well as other general books, pamphlets, videos, and other educational materials on futures trading. Current minimum margin requirements and recent price information are available at the websites. Before trading futures, you should contact the exchanges that trade the contracts in which you have interest, and obtain any and all information they have to offer.

At the end of Table 6.3 are the addresses of some other agencies and organizations of interest. The CFTC, created on April 21, 1975 pursuant to the Commodity Futures Trading Commission Act of 1974, regulates all futures markets and futures trading in the United States, although it should be noted that the industry is largely self-regulated by the exchanges themselves. CFTC approval is required for all new contracts, contract terms, trading rules, margin requirements and so on. The CFTC enforces the rules that protect futures traders' funds. The CFTC also offers publications on futures trading, many of which are free.

The National Futures Association (NFA) began operations in October 1982. It serves as the futures industry's self-regulatory organization. The NFA has responsibility for any regulation that the CFTC believes should be handled internally by the futures industry itself. For example, the NFA establishes ethical codes for the industry, considers FCM–customer disputes, screens new FCMs and other futures markets participants that deal with the public, and disciplines its members when necessary.[16]

6.7 The Purposes of Futures Markets

As Figure 1.2 showed, futures trading has exploded since the early 1980s. An interesting question is: Why do futures contracts exist? Do they serve any useful purpose, or are they merely gambling instruments that have created speculation-based price volatility? Since the stock market crash of 1987, the scandals in the trading pits of the late 1980s, and financial debacles such as those involving Procter & Gamble and Barings in the 1990s, many critics have questioned the raison d'être of futures and have called for their abolition. Indeed, since the first appearance of futures and forward contracts, there have been lengthy periods during which they were banned.

Futures markets exist for several reason:

a. Futures make transactions across time easier. Production, consumption, and inventory decisions can be more optimally made when futures markets exist. They allow individuals to *quickly* create *low cost* agreements to exchange money for goods at future times. Both speculators and hedgers will transact in the futures markets when speed is important. The transactions costs of trading futures are minuscule relative to the dollar amount of the commodity underlying most contracts.

b. Futures allow informed individuals to act on their superior information. In this way, prices will become more efficient. In other words, prices will reflect more information, and therefore resources will be allocated in ways that are closer to optimal.

c. Futures allow individuals to hedge against undesirable (costly) price changes. Producers and users can then do what they do best: produce and use. They should not be in the business of price speculation. Hedgers transfer price risk to speculators. Price volatility and uncertainty are major preconditions for a successful futures contract, attracting both speculators and hedgers.

d. Futures prices contain information. This is called the "price discovery" function of futures. Producers and consumers can get an efficient idea of what the future spot price will be, and what future supply and demand of a good will be, by observing the current futures price. In so doing, they can make better production and storage decisions.

6.8 READING FUTURES PRICES IN *THE WALL STREET JOURNAL*

6.8.1 Commodity Futures

Figure 6.4 is a listing of price data for commodities such as agricultural goods, metals, and energy products. The data come from the July 29, 1999, *Wall Street Journal*, summarizing trading activity on July 28, 1999. Price data for selected financial futures are presented in Section 6.8.2.

The header line of each commodity makes note of the commodity type, the exchange on which the futures contract trades, the quantity of the good underlying one contract, and the units of the price. For example, the first entry gives price data for the corn futures contract that is traded on the Chicago Board of Trade (CBT). One futures contract covers 5000 bushels of corn. The prices are in cents per bushel. A trader can and should contact the exchange for details on the quality of corn that is deliverable, the locations to which the corn must be delivered, and the delivery procedure.

Below each header line are the different delivery months for each contract. For corn, there are futures contracts for delivery in September and December 1999, and March, May, July, September, and December 2000. There may be other delivery months, but volume is insufficient for the *Wall Street Journal* to provide price data. One *very* important question is not answered: What day of the delivery month is the delivery day? Or are there a range of possible delivery dates? Related to these concerns is the absence of a listing for the last day of trading. It is even possible that the individual who is short the contract can initiate delivery in the month before the delivery month. For example, the last trading day for the NYMEX heating oil and unleaded gasoline futures is the last business day in the month preceding the delivery month (August 31, 1999, for the September 1999 contracts); the crude oil contract terminates trading even earlier, in the month preceding delivery (note the natural gas futures for Oct. 2001–June 2002 delivery; it ceased trading on the third business day prior to the 25th calendar day of the month preceding the delivery month, which was July 21, 1999).

Eight items are printed for each delivery month.

1. The **open** is the first price at which a contract traded. The Sept. 1999 corn contract, for instance, opened at 197 cents ($1.97) per bushel. The value of corn underlying this contract of 5000 bushels is therefore $9,850 ($1.97/bu × 5000 bu).

2, 3. The **high** and the **low**, respectively, are the highest and lowest prices at which a contract traded on the preceding day.

4. "**Settle**", the settlement price of 196 cents/bu, determines the mark-to-market cash flow for the day. It is representative of the prices at which contracts traded during a specified closing period, frequently the last minute of trading. If no trades occurred during that closing period, then the exchange selects a representative price that it feels reflects market conditions. That is why more distant contracts frequently all change by the same amount in Figure 6.4 (note the natural gas futures for Oct. 2001–June 2002 delivery).

5. The **change** is the change in settlement prices from the preceding day (Tuesday, July 27, 1999) to the current day (Wednesday, July 28, 1999). Corn for delivery in September 1999 fell by 0.5 cent per bushel, from 196½ to 196 cents per bushel. Tuesday's settlement price can only be inferred, using Wednesday's close of 196, and the change of ½ a cent. The value of corn underlying the contract therefore fell by $25, and all accounts that were short the Sept. 99 corn futures contract were marked to market with a profit of $25.00. Likewise, all long positions were debited (a loss) $25.00.

6, 7. The **lifetime high** and **low**, respectively, are the highest and lowest prices at which each contract has traded, since trading began. The Sept. 99 corn contract first began trading more than

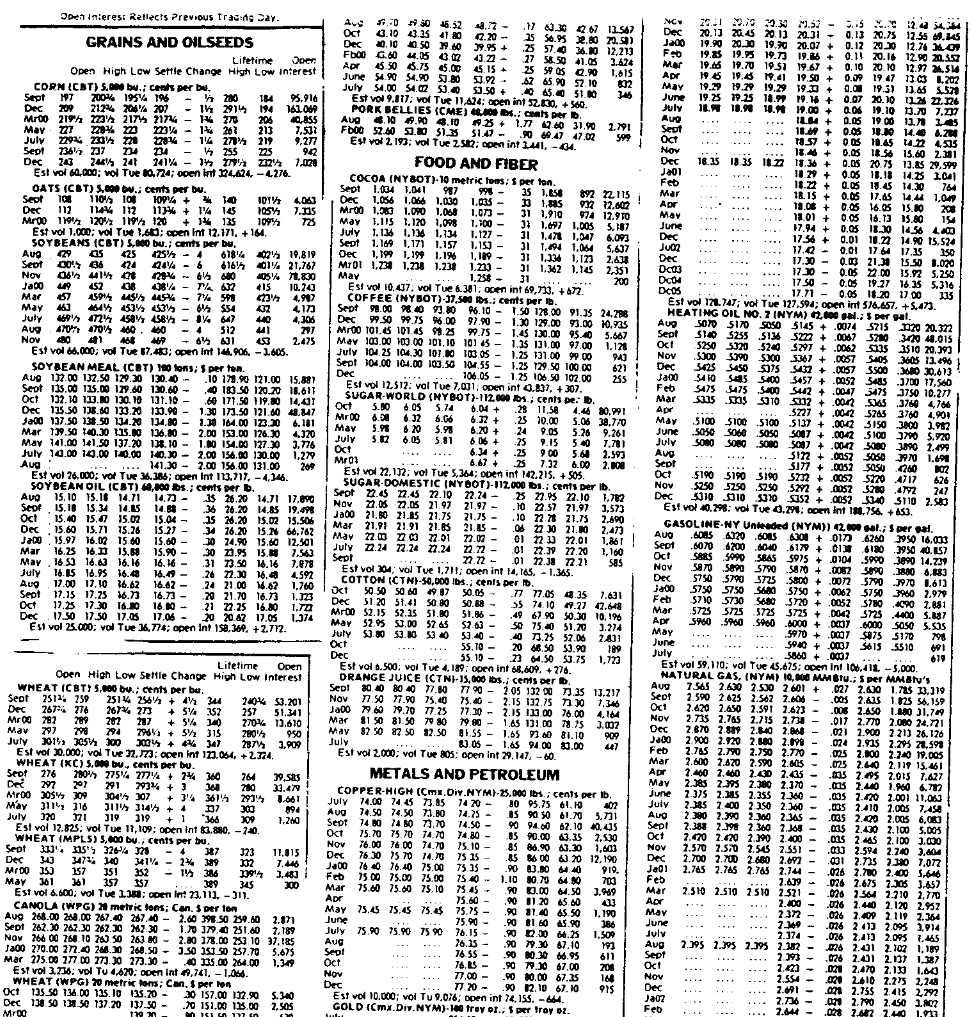

Figure 6.4 Price data for selected financial futures. (Reprinted with permission of *The Wall Street Journal*, © July 29, 1999.)

a year ago, and at one point in its life it traded as high as 280 cents/bu. At another time, a contract traded for as little as 184 cents/bu.

8. The **open interest** is the number of futures contracts outstanding. There are 95,916 Sept. 99 corn contracts outstanding: 95,916 long positions, and 95,916 short positions. Open interest may or may not change when two traders offset their positions in a trade. If you are long one Sept. 99 corn contract, and you sell it to a trader who previously had no position in this contract, then open interest remains unchanged. If, however, you sell it to an individual who was previously short a Sept. 99 corn contract, then open interest declines by one contract.

Below the price information for all the months is the volume on the two most recent trading days; the most recent day is estimated and the preceding day (July 27, 1999) is actual volume. Finally, the total open interest, for all delivery months, is given. There were 4276 fewer corn futures contracts outstanding (for all delivery months) on July 28 than on July 27.

The value of the good underlying the contract can be computed by multiplying the number of units of the good by the price. Because all contracts are marked to market, you can also multiply the contract size by the change in the futures price to determine the cash flow resulting from the day's trading.

EXAMPLE 6.3 If you were long the December 1999 copper contract, then according to Figure 6.4 you realized a mark-to-market loss of \$212.50 (+\$.0085/lb × its contract size of 25,000 lb). If you were short the October 1999 natural gas contract, your profit was \$80 (the futures price fell by \$.008/MMBtu × 10,000 MMBtu).

6.8.2 Financial Futures

The following is a brief discussion of how to read and interpret *Wall Street Journal* price data for several futures contracts on financial assets and indices, which are shown in Figure 6.5. The data are from the *Wall Street Journal* of January 17, 2001, summarizing transactions made on January 16. Chapters 9 and 10 present more detailed information on Treasury bond, Treasury bill, and Eurodollar futures.

The first set of contracts shown in Figure 6.5 consists of the interest rate futures. The first contract listed is the long-term Treasury bond futures contract, which trades on the Chicago Board of Trade. This contract calls for the delivery of \$100,000 face value of long-term (about 20 years to maturity) Treasury bonds. The futures prices are given in "points and 32nds of 100%." Think of this as a percentage of \$100,000 of Treasury bonds. For instance, the settlement price for the March 2001 contract was 103-06. This is 103 and 6/32% of \$100,000; 6/32 equals 0.1875. The value of the bonds underlying the contract is therefore 1.031875 times \$100,000, which is \$103,187.50. The value of the bonds underlying the June 2001 Treasury bond futures contract, given its settlement price of 102-30, is \$102,937.50 (102 and 30/32 percent of \$100,000).[17]

The March and June 2001 Treasury bond futures contracts closed 9/32 (9 ticks) above the previous day's settlement price. In other words, the March contract closed at 102-29 on January 15, 2001, and the June contract closed at 102-21. The bonds underlying the contracts on January 15

Figure 6.5 Financial futures prices for January 16, 2001. (Reprinted with permission of *The Wall Street Journal*, © January 17, 2001.)

were thus worth $102,906.25 and $102,656.25 for March and June delivery, respectively. From this, we see that one tick equates to $31.25 in value (e.g., $103,187.50 minus $102,906.25 for the March contract is $281.25, which is the value of a rise in price of 9 ticks; $9 \times \$31.25 = \$281.25$).

The next contract in the first column of Figure 6.5 is the actively traded Treasury note futures contract, which calls for the delivery of treasury notes with about 6.5–10 years to maturity. Further down are the five-year and two-year Treasury notes contracts. Prices for these three contracts are presented slightly differently than the Treasury bond contract. The (10-year) Treasury notes and the five-year Treasury notes contract actually trade in halves of 32nds, and the two-year Treasury notes contract trades in quarters of 32nds. When prices are given in halves or quarters of 32nds, the *Wall Street Journal* drops the "1" from any price exceeding 100 and adds either a "2" (for 1/4), a "5" (for 1/2), or a "7" (for 3/4) to the last digit of the price. Thus, the closing price for the March Treasury notes contract was actually 104 and 16.5/32, but it is presented as 04-165. Similarly, the closing price of the March two-year Treasury note contract was 101 and 29.25/32, but it is presented as 01-292. The smallest tick size for these three Treasury note contracts is $15.625. Note that the size of the two-year contract is $200,000.

Treasury bill futures contracts trade on the Chicago Mercantile Exchange. They call for the future delivery of $1 million face value of 13-week Treasury bills. In Figure 6.5, the March futures settlement price of 95.10 is actually 100 minus the futures interest rate. In other words, given the futures discount yield of 4.90%, the futures price is then $100 - 4.90 = 95.10$. Each tick represents one basis point, and it equals 0.01, which equals $25. On the prior trading day, January 15, 2001, the futures price was two ticks higher, or 95.12. Therefore, all shorts profited by $50, and all longs lost $50. The discount yield, shown in the next column, is an interest rate that is used for Treasury bills. This futures discount yield is essentially a forward rate, just like you learned about in Chapter 5. The March futures discount yield of 4.90 is a forward three-month rate that exists for delivery on some day in March 2001 (i.e., from some date in March until a day in June 13 weeks later).[18] Buying a March Treasury bill futures contract at a futures price of 95.10 commits you to purchase $1 million (face value) of three-month Treasury bills in March, at a price that is consistent with a discount yield of 4.90%.

The second most actively traded futures contract in the world (at least during the first 11 months of 2001) is the Eurodollar time deposit futures contract, which trades on the CME. Like the Treasury bill contract, each tick (0.01) equals $25. Figure 6.5 shows that the February contract rose in price by two ticks, which reflects a two-basis-point decline in the three-month Eurodollar futures interest rate. If a Eurodollar futures price were to fall by *one point*, say, from 94.45 to 93.45, this would reflect a rise in interest rates of 100 ticks, or $2500. This is equivalent to a rise in the futures yield of Eurodollar time deposits of 100 basis points, from 5.55% to 6.55%. Note the huge open interest of most Eurodollar futures contracts. Also note that the delivery dates stretch out all the way to December 2010, almost 10 years into the future. The December 2010 futures yield of 7.30% is a forward rate that exists for a three-month period beginning in December 2010.

Futures on other Eurocurrencies, such as the Euroyen, the Euribor (for the new currency, the Euro), and the Euroswiss also are actively traded, as are foreign bond futures such as Euro-denominated government bond contracts. You are encouraged to contact the exchange on which the contracts trade to learn the current information concerning any contract you trade.

The next section of contracts shown in Figure 6.5 are currency futures, which require the future exchange of different foreign currencies. For example, the Japanese yen futures contract calls for the delivery of ¥12.5 million. The yen futures prices drop off the first two zeroes, which is why you see

(.00) on the header line for the yen contract in Figure 6.5. The March futures price for one yen is actually \$0.008596. Multiply this futures price by ¥12.5 million, and you obtain the value of JPY underlying one futures contract. For March delivery, this equals \$107,450 (¥12,500,000× \$0.008596/¥).

On January 16, the March yen contract closed \$0.000063/¥ higher than its settlement price on January 15. Because **one tick** is the term used to denote the minimum price change that a contract can make, this represents a rise of 63 ticks, from \$0.008533/¥ to \$0.008596/¥. Alternatively, the value of the yen underlying the contract rose from \$106,662.50 to \$107,450, or \$787.50. Since the rise of 63 ticks increased the value of yen underlying the contract by \$787.50, it follows that one tick equals \$12.50 (\$787.50/63 ticks = \$12.50/tick).

The last currency listed is the Euro (€), and the March contract for euros fell by 90 ticks on January 16. One euro futures contract covers €125,000. Therefore, an individual who was long one March euro futures contract lost lost \$1125 (90×\$12.50). This occurred because the value of the euros underlying the contract fell from \$118,987.50 (€125,000×\$0.9519/€) to \$117,862.50 (€125,000×\$0.9429/€).

You should be able to analyze the price data for the other foreign exchange futures contracts by reading the header line concerning the size of the contract (in units of foreign currency), and the price rule (dollars per unit of foreign currency).

In the last section of Figure 6.5 are prices of index futures, most of which are stock index futures. The most popular stock index futures contract, in terms of open interest, is the S&P 500 Index, which trades on the Chicago Mercantile Exchange. The underlying asset of this contract is effectively 250 shares of the S&P 500 Index, which is itself a portfolio of stocks. The value of the stock underlying the contract equals \$250 times the futures price. In Figure 6.5, we see that the settlement price for the March 2001 S&P 500 futures contract was 133550 or \$1335.50. Therefore, the value of the stock underlying the contract was \$333,875 (1335.50×250).

In the "Change" column, the change in the futures price is given. On January 16, 2001, the settlement price of the March S&P contract was 5.20 points above the January 15th settlement price. This means that anyone who was long one S&P 500 futures contract profited by \$1300 (5.20×250) as a result of daily resettlement. Short positions lost \$1300 per contract. One point equals \$250. The value of the stock underlying the March contract on January 16 was \$333,875 (1335.50×250). On January 15, the value was \$332,575 (1330.30×250). The increase in stock value underlying the March contract, from January 15 to the next trading day of January 16 was \$1300 (\$333,875 − \$332,575, which equals 5.20×250).

Each *tick*, or 0.10 of stock index futures price movement, equals \$25 (0.10×250). A tick is the smallest unit by which the price of a S&P 500 futures contract can change. Tick sizes differ for different futures contracts. All stock index futures contracts are cash settled. On the last trading day, the final settlement price is set equal to the spot S&P 500 Index value, which is in units of 0.01. Thus, the size of the last tick for the S&P 500 contract is actually 0.01. Cash settlement means that there is one last mark-to-market cash flow that occurs on the last day of trading, and then the contract ceases to exist. There is no delivery of the underlying asset.

The highest price that the March 2001 S&P 500 contract ever sold for, during the time that it has traded (it has traded for more than a year) was \$1642.60. The lowest futures price was \$1270.00. At the close of trading on January 16, there were 468,147 contracts for March delivery outstanding (open interest). Below all the price data is the estimated January 16 trading volume of 66,963 contracts, and the actual January 15 trading volume of 72,792 contracts. Finally, the *Wall Street Journal* presents the preliminary (estimated) high, low, and closing values for the *spot*

S&P 500 Index (Idx prl). On January 16, the spot index closed at 1326.65, and this was 7.83 points below its close on January 15.

6.9 Limits on Price Fluctuations

As mentioned earlier, every futures contract has a minimum size for trade-to-trade price changes, called a tick. For example, the tick size for the wool futures contracts that trade on the Sydney Futures Exchange is one Australian cent/kg, or 25 Australian dollars (denoted A$25).

Many futures contracts also have maximum limits on daily price changes, which define the maximum amount by which a price can change in one day. Note that the exchanges will frequently change these limits when they feel it is appropriate or necessary, and the price limit may differ for the most nearby contract from contracts with delivery in more distant months.

The purpose of such price limits is to allow the markets time to digest new information and dampen possible tendencies to overreact. It also may provide an incentive for losers to meet their margin calls (the holder of a losing futures position may meet each of a series of small margin calls, but default on one very large margin call). But the drawback to these daily maximum price change limits is that they also may prevent traders from closing out or opening a desired position. For example, suppose that a revolution in a Middle Eastern country caused the price of oil to rise by $45/bbl. With no price limits, the futures price of crude oil would immediately rise by about $45/bbl. But the daily limit on the NYMEX crude oil futures contract is $15/bbl. Therefore, it would take three trading days during which no trading occurs to reach a price at which futures trades would be made. Today's futures price would immediately rise by $15/bbl (a *limit up* day), and there would be an enormous quantity of buy orders at that price, but no sell orders. The same would happen tomorrow and the next day, for a total of three trading days.

Some believe that without price limits, futures price changes would sometimes be irrational. Daily price limits, in the opinion, serve the useful purpose of allowing markets the time to gather more information, and more accurately assess the implication of the news.[19] (Others believe that no price change can ever be judged to be irrational.)

6.10 Orders and Position Limits

All orders to trade futures contracts have the following features in common:

Whether the trader wishes to buy or sell

The name of the commodity

The delivery month and year of the contract

The number of contracts

The exchange on which the contract trades (if it trades on multiple exchanges)

Whether it is a market order or a limit order, if the latter, it must include a stated limit price at which the trade should be made

Whether the trade is a day order (if it is not filled by the close of trading, it is canceled) or a good-til-canceled order

If a trader places a **market order** to buy or sell, she will assume a long or short position at the price that is prevailing in the futures **pit** at that time.[20] Upon receiving such an order, a representative of the trader's FCM will go to the pit where that contract trades and try to get the best price possible, based on supply and demand conditions at that moment. There is risk that in a "fast" market, the trade price will be quite different from the one quoted to the customer by the FCM just a minute or two earlier.

Limit orders to buy or sell specify that the trader wishes to trade at the stated price or better. The risk here is that the trade will not be executed. **Market-if-touched** (MIT) orders are like limit orders, except that they become market orders once a trade at the specified price has occurred. For example, an MIT buy order for gold with a specified price of $273.40 becomes a market order as soon as a trade occurs at that price or lower. The trader may get that price, or one above or below it. In a rapidly declining market, the sequence of trades after placing the MIT buy order might be $275, $272.60, $270.80. The trade should be filled at whatever price occurs **after** the trade at $272.60.[21] In this sequence, it should thus be at $270.80.

Stop orders, also known as **stop-loss-orders**, also specify a price. There are two types of stop order: to buy and to sell. A stop order to buy is usually placed by a trader who is currently short the contract and desires to limit his losses should the price start rising.[22] Thus, stop orders to buy are placed at a specified price that is above the current price, and they become market orders when the futures price is at or above the specified stop price. Stop orders to sell are placed at a specified price that is below the current futures price.

In contrast to a stop order to buy, a limit order to buy would specify a limit price *below* the current price. In this case the trader wishes to initiate a long position at a futures price somewhat more favorable (lower) than the current one, or would be satisfied with the profits on an existing short position if the futures price should decline to the specified price.

A **stop limit order** becomes a limit order once the specified price has been reached. Thus, in Example 6.4, if a trade takes place at $300/oz., a stop limit buy order would result in the trader offsetting his short position only at $300/oz. or better (a price less than or equal to $300/oz.). A stop limit order can even specify a stop price different from the limit price. For example, a trader might place a stop limit order to buy back the gold contract, with a stop of $300 and a limit of $297. This means that once there is a trade at $300, the trader will offset his short position, but only at prices between $297/oz. and $300/oz. It is important to note that a trader using stop limit orders has the risk of an order going unfilled in some fast market conditions.

There are many other types of order that can be placed at one exchange or another, and some of these are listed in Table 6.4. Traders should inquire what order types are accepted at exchanges on which contracts of interest trade. For example, MIT orders are accepted at the CME, but not at

EXAMPLE 6.4 A trader initially sells a gold futures contract at $290/oz. After that order has been filled, he can place a stop order to buy at $300/oz. to control his losses if it develops that he was wrong in his belief that the price of gold would fall. If a trade takes place at $300/oz., then his order becomes a market order to buy, and he will have his short position offset at the next price, whatever that may be.

TABLE 6.4 Other Types of Futures Orders

1. A **one-cancels-the-other** order consists of two orders placed simultaneously, but as soon as one order is executed, the other is canceled. For example, the current gold futures price for June delivery might be $300/oz. A trader who is long a contract might place the following one-cancels-the other order: "sell one June gold futures contract $305 limit/$295 stop." If the price of gold rises to $305, the long position is offset, profits are taken, and the $295 stop order is canceled. Alternatively, if gold's price declines to $295/oz., the trader cuts his losses and the $305 limit sell order is canceled.

2. A **spread order** specifies two trades that must be filled together. The order can specify a difference in prices, or it can be a market order. For example, a trader who places an order to buy one Eurodollar futures contract and sell one Treasury bill contract might specify a spread price of 85 ticks "to the sell."[1] Suppose that the Eurodollar futures price for September 1999 delivery is 94.53, and the September Treasury bill contract price is 95.33 (see Figure 6.5). Thus the current spread is 80. If the spread widens to 85, the trader wants to speculate that the spread will subsequently narrow. Put another way, he believes that once the spread reaches 85, the Eurodollar futures price will then rise more than the Treasury bill futures price. This is an example of an *intercommodity spread*, or *intermarket spread*. Another type of spread is the *time spread*, or *calendar spread*, in which a trader buys futures on a good for delivery in one month and sells futures on the same good for delivery in another month.

3. A **fill-or-kill order** specifies a limit price, usually close to the current market price, and if the order is not filled (executed) immediately, it is canceled.

4. A **contingent order** places two orders, but only one is entered at the exchange initially. As soon as that order is executed, the second one is entered. Thus, a trader can place an order to sell a crude oil futures contract at 20.60 and, contingent on the execution of that order, buy (offset) the same contract at a limit price of 20.00.

5. A **market-not-held order** is also known as a DRT (disregard tape) order. In this type of order, the trader relies on the skills of the FCM's representative in the pit to decide if and when to fill the order. If the broker in the pit believes he can get the trader a better price in the next few minutes, he will delay executing the order.

6. An **on-the-close** order is one that will be filled during the last minute or so of trading. More common are "limit or market on close" orders, which specify a limit price for the day, but if the order is not executed by the close of trading, it becomes a market order.

[1] If the Treasury bill futures price is above the Eurodollar futures price, and the Treasury bill contract is being sold, the phrase, "to the sell" is used.

the CBOT. Many FCMs can also supply a summary of orders that can be placed at the different exchanges.

There are often limits on the net long and net short positions that any one speculator (or associated group of speculators) can assume in the futures contracts of a given commodity. The Commodity Futures Trading Commission passed the rule that the exchanges impose such position limits after the crisis caused by the Hunt family's attempt to corner the silver market in 1979–1980. The Hunts had a long position in over 18,000 silver futures contracts, and at the same time owned much of the supply of the metal itself. Thus, anyone who was short silver futures contracts would have had trouble finding silver to deliver because the Hunts owned most of it. Position limits, however, do not exist for hedgers.

The position limits themselves differ from commodity to commodity. For example, as of June 2000, the position limits for CME S&P 500 futures and NYMEX light sweet crude oil futures were 20,000 contracts each; for CBOT two-year Treasury notes the position limit was 5000 contracts. There were no position limits for Eurodollar futures or Euro FX currency futures. Often traders with large positions in any one type of contract must provide accountability information on the nature of his position, his trading strategy, and hedging information to the CFTC and/or the exchange. For more discussion on position limits for exchange traded futures and options, see the CFTC's website (www.cftc.gov/opa/backgrounder/opaspeclmts.htm).

6.11 Individuals in the Futures Industry

In this section, we describe the different types of individual who trade futures, who also play a part in executing trades and determining futures prices.

Besides individuals on the floors of the exchanges, there are usually three broad classes of futures traders: speculators, hedgers, and arbitrageurs.

6.11.1 Speculators

Speculators assume risks, and their goal is to profit from price fluctuations. They may be long or short futures, or they may hold spread positions. Speculators themselves can be subdivided into two subgroups: position traders and day traders.

Position traders enter a position and hold it for several days, weeks, or months. They may use **technical analysis** to develop beliefs about future price movements. Technical traders primarily study price charts to discern trends that they believe will persist. Other position traders employ **fundamental analysis** to form their opinions, using macroeconomic data to develop their predictions about future price movements.[23]

Day traders speculate only about price movements during one trading day. They never go home holding a futures position, and therefore probably sleep better at night. Day trading is costly because prices must be monitored closely throughout the day. Day traders are likely to open and close positions several times during the day, with the goal of making a profit of just a few ticks per trade.

6.11.2 Hedgers

Hedgers initially (before they hedge) are exposed to the risk of a price change. They may initially be long or short a good and would therefore experience losses if prices were to move against them. For example, an oil trading company might purchase a large amount of crude oil to import to the United States. This transaction exposes them to the risk that during the week it takes to transport the oil to the United States, oil prices will fall, and the oil will have to be sold at lower prices. This firm can sell crude oil futures contracts to hedge. If oil prices do decline, the trading company will lose money on the inventory of oil (the spot position) but will make money on the futures contracts that were sold. This is an example of a **short hedge**.

Another company might be short the good, and therefore fearful that prices will rise. This company will enter a **long hedge** by going long futures. For example, a jewelry manufacturer might sign a contract in January 2000 agreeing to deliver gold jewelry to a retailer in September 2000 at a fixed price. The manufacturer does not have the gold in inventory (the gold would have to be insured; the manufacturer may not have sufficient warehouse space, or not enough cash to buy the gold today, etc.) and does not wish to purchase the gold until May 2000, when manufacture of the jewelry requiring that gold is scheduled to begin. The manufacturer is exposed to the risk that gold prices will rise between January and May. If prices rose, it would have to pay more for its raw material. Since the price to be received for the output is already fixed, the manufacturer should hedge by buying gold futures contracts.

The use of futures in risk management will be the subject of Chapter 7.

6.11.3 Arbitrageurs

An arbitrageur profits by observing that a good, or equivalent goods, sells for different prices in two different markets. The cost-of-carry pricing model, covered in Chapter 5 for forward contracts, is the arbitrage mechanism that sets the relationship between spot and futures prices. Sometimes temporary supply and demand disturbances lead to mispricing. When this occurs, arbitrageurs will buy cheap futures and sell the overpriced spot good, or sell overpriced futures and buy the spot good.

6.11.4 Individuals on the Floor of an Exchange

The people who trade on the floor of the exchange can be categorized by their goals and/or functions. Statistics concerning the numbers of individuals who fall into these categories, as of February 2001, can be found at the National Futures Association (NFA) website (www.nfa.futures.org/registration/nfa_membership.html).

Floor traders are exchange members who trade futures contracts in the futures pit. A floor trader who trades solely for his own account is called a **local**. Locals provide liquidity for the market, usually by operating as **scalpers**. A scalper is a very short term trader. Basically, scalpers want to buy futures at the bid and quickly turn around to sell it at the asked.[24] As of September 1998, 1364 floor traders were regulated by the CFTC. Another class of futures markets participants consists of the **floor brokers**, who execute trades for other parties such as FCMs. Floor brokers earn income by charging small fees, perhaps a dollar or two per contract, to the parties for whom they execute trades. There were 9538 floor brokers registered with the CFTC in 1998.

An important regulatory debate emerged in 1989 about floor traders who operate as both locals and floor brokers. Such practice, called **dual trading**, had been abused by some traders. Basically, a floor trader who made two contract purchases in a short period of time at two prices, one for a customer and one for himself, had the "ethical dilemma" of declaring which contract he bought for himself and which he bought for a customer. A 1989 CFTC study concluded that fewer 10% of all floor traders practiced dual trading, and few of the dual traders cheated their customers. However, the study also concluded that, in general, the practice contributed only slightly to the liquidity of futures markets. Dual trading is still permitted today, but there are extensive audit trail procedures and record-keeping systems that monitor the practice and constrain abuse by floor traders.

6.11.5 Other Market Participants

The annual Source Book, published each year in January by the magazine *Futures*, is a good source for some specific names and addresses of the following types of firms and individuals.

We have already mentioned **futures commission merchants**, FCMs, who are the brokers of the futures industry. There are full-service FCMs such as Merrill Lynch and Dean Witter, and discount FCMs such as Lind-Waldock, Jack Carl Futures, and First American Discount Corp. As of February 2001, there were 191 FCMs registered with the CFTC. A list of registered FCMs is available (www.cftc.gov/tm/tmfcm.htm).

Associated persons, APs, are the individuals who work for FCMs in soliciting and accepting orders. Most APs are the futures industry's equivalent to the stock brokerage industry's account executives. In February 2001, there were 47,565 APs registered with the CFTC.

Commodity trading advisers, CTAs, analyze futures markets and give trading advice to anyone who wishes to pay for it. Anyone who makes recommendations to buy or sell futures, develops trading systems, or provides information about commodities must register with the CFTC as a CTA. In February 2001, there were 919 CTAs registered with the CFTC.

Introducing brokers, IBs, are individuals who direct business to FCMs and CTAs but themselves are not APs or CTAs. An IB might solicit orders and take orders, but once received, the orders are passed on to an FCM for execution. Other IBs might pool funds of small investors into a large amount so that they can gain access to a successful CTA who has a large minimum investment requirement. The IB registration category was introduced by the CFTC in 1982. In 2001 there were 1400 IBs registered with the CFTC.

Commodity pool operators, CPOs, are the futures industry's equivalent to mutual funds. They accept money from investors. The monies are pooled and used to trade futures. Many CPOs hire CTAs to make the trading decisions for them, or at least to aid them in making decisions. In 2001 there were 1455 CPOs registered with the CFTC.

Many firms analyze the performance of CTAs and CPOs. Some of the performance data of **Managed Account Reports**, which is one such "CTA tracker," are published monthly in *Futures*. Academic research has been critical of the performance of CPOs (Irwin and Brorsen, 1985; Elton, Gruber, and Rentzler, 1987; Cornew, 1988; Edwards and Ma, 1988; Irwin, Krukemyer and Zulauf, 1993). These studies have found that CPO returns are negative on average: fewer than 50% of them produce positive returns. Moreover, they have very high expenses, and their prior performance is of little value in predicting future performance.

6.12 Taxes and Commissions

6.12.1 Taxes

The tax code for hedgers who trade futures differs from the laws that define speculators' tax situations. Hedgers should consult a tax accountant or a tax attorney for advice.

Section 1256 of the IRS code requires that individual speculators mark to market all futures positions for tax purposes on the last trading day of the year. For example, suppose that on November 24, 2000, you sold one gold futures contract for February 2001 delivery at the settlement price of $290/oz. On December 31, 2000, the settlement price for the February 2001 gold contract is $267.40. You offset your position on January 16, 2001 at a futures price of $264.

The tax law requires you to mark to market your position on December 31, 2000, and declare the profit (or loss) for tax purposes. Thus, if capital gains are taxed as ordinary income, your taxable income for 2000 will be $2260 greater than had you not traded the gold futures contract. If you are in the 28% marginal tax bracket, your tax liability will be $632.80 greater. Note that you are realizing this income, eventhough you have not offset your position.

Then, the IRS establishes a new basis[25] for your trade: $267.40. In 2001 you will report a profit of $340 ($26740 – $26400) on the futures trade.

Generally, 40% of the gain or loss is treated as a short-term capital gain or loss, and 60% is treated as a long-term capital gain or loss.

Before 1981, tax straddles were a legal means of deferring taxes. In a tax straddle, a trader would select a volatile futures commodity, buy some contracts for delivery in one month of the following year, and also sell an equal number of contracts for delivery in an adjacent month of that year (a calendar spread). With a volatile commodity, it would then be likely that in late December of the current year, the trader would have a sizable gain on the contracts for one of the months, and a roughly equal loss on the contracts for delivery in the other month. The trader would offset the losing position, thereby realizing a short-term capital loss, which could be used to reduce the tax liability in the existing year. Taxes on the profitable position would be deferred to the next year. The Economic Recovery Tax Act of 1981 ended this means of deferring taxes. Moreover, subsequent tax legislation has since ended the distinction between long-term and short-term capital gains and losses for tax purposes.

See Conlon and Aquilino (1999) for detailed information about the taxation of futures and all other derivatives.

6.12.2 Commissions

Commissions are paid only when futures trades are offset, or end in delivery or final cash settlement. Full-service FCMs charge about $100 to trade one contract (round turn). The per-contract commission declines as more contracts are traded. Some offer discounts for day traders, and some offer discounts to traders who only desire execution, not advice. Discount FCMs might charge between $15 and $40 per contract when the position is closed. Again, the commission per contract can decline with larger trades. In addition, day trader discounts are often offered, and discounts may be available to traders who have access to their own futures price reporting systems. Larger traders such as corporations, banks, and professional trading firms, may pay $10/contract or less. Finally, floor traders on the floor of the exchange pay as little as $1.50/contract to trade.

Sophisticated traders will be willing to pay more in commissions if they feel that they are getting better execution in the pits. An extra few dollars per contract in commissions is a small price to pay if the FCMs representatives can save the customer a tick on a sizable fraction of the trades. Active traders can often get a feel for their FCMs execution abilities after a while.

6.13 Summary

This chapter provided an introduction to futures markets and trading. For novices, futures may seem confusing. Each futures contract has its own set of rules. For this reason, you should initially select just one type of futures contract to learn about before ever trading it.

First, the differences between forward contracts and futures contracts were covered. Perhaps the most important difference for valuation purposes is the margin requirements and daily resettlement (mark-to-market) features of futures. The concepts of basis and convergence were introduced.

Then the different types of futures contract were outlined. Some regulatory issues were discussed. You learned how to read price data presented in the financial press. The different types of order that can be placed were summarized.

The economic rationale for futures trading was briefly discussed. The different individuals who make up the futures markets were categorized. Finally, the chapter provided a brief discussion of taxes and commissions.

The futures exchanges supply a great deal of additional information on futures trading. You might begin by accessing their websites.

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Notes

[1]Many futures contracts have delivery options inherent in them. For example, a contract may specify a range of goods of different quality that can be delivered (Kamara and Siegel, 1987; Hemler, 1990; Chance and Hemler, 1993). Or it may offer several locations to which the goods can be delivered, or several dates on which delivery can be made. It is interesting that the *short* position gets these options. For example, if the futures contract offers a range of delivery dates, the short decides when to make delivery. This differs from American option contracts, in which the owner (the long position) of the option decides when to exercise it.

[2]A clearinghouse is an agency associated with one or more exchanges. Its functions typically include the following: (a) to match, process, register, confirm, settle, reconcile, and/or guarantee trades, (b) to become a party to every trade, so as to (nearly) eliminate credit risk, (c) to operate the mark to market process (collecting and paying variation margin), and (d) to handle the delivery process. A clearinghouse might be a subsidiary of an exchange, or an independent entity. See Bernanke (1990) and Baer, France, and Moser (1995) for further details on the clearing process.

[3]A futures commission merchant (FCM) is the futures industry's equivalent to a stockbroker. The importance of the solvency of a customer's FCM is stressed because a case study and theoretical analysis by Jordan and

Morgan (1990) shows that the clearinghouse in reality guarantees only that its members will be paid. Customers of a failed FCM do bear default risk.

[4]Bates and Craine (1999), however, assess the probability that a major clearinghouse could have failed during the 1987 stock market crash. At that time, there were rumors and fears that a clearinghouse could fail, and the Federal Reserve had to assure markets that the central bank would supply liquidity to the system if needed.

[5]See Jones (1982) and Garbade and Silber (1983) for discussion concerning the economics of cash settlement. Cash settlement is desired when delivery costs are large (e.g., with the S&P 500 Index futures contract), when manipulation is possible by traders who might corner the available supply of the deliverable asset, and/or when the futures price is based on an index. However, Lien (1989a, 1989b) and Cita and Lien (1992) cite several problems with cash settlement, and these are documented in a study of the municipal bond futures contract by Cornell (1997).

[6]As discussed in Chapter 3, the two parties *could* negotiate good faith money, or collateral, to increase the likelihood that each will abide by the terms of the contract, and therefore lower default risk. Each party would then deposit some amount of money, securities, or other assets with a neutral third party.

[7]Recently, it has become more popular to refer to "margin" as "performance bond." Thus, there is an initial performance bond and a maintenance performance bond.

[8]Further discussion on SPAN can be found at the websites of the Chicago Mercantile Exchange (www.cme.com) and the London International Financial Futures and Options Exchange (www.liffe.com). Kupiec (1994) explains why SPAN was developed (in response to the 1987 stock market crash), and how S&P 500 futures margins have been set under SPAN.

[9]For example, in early 1980 the tremendous volatility in silver prices led the COMEX to increase the initial margin requirement to $75,000/contract, equal to about 30% of the value of the silver underlying the contract. In contrast, the initial margin requirement of $2400 in April 1989 was only about 8% of the value of silver in the 5000 oz. contract. For more details on the Hunt brothers' attempt to corner the silver market in 1979–1980, see Paul Sarnoff (1980), Fay (1982), an October 1982 report by the Securities and Exchange Commission, and articles in the *Wall Street Journal* and *Barron's* in early 1980.

[10]Several papers have studied the economic theory behind margin requirements, and their effects on trading. See Anderson (1981), Burghardt and Kohn (1981), Telser (1981), Kahl, Rutz, and Sinquefield (1985), Fishe and Goldberg (1986), Hartzmark (1986), Fishe et al. (1990), Kalavathi and Shanker (1991), Pliska and Shalen (1991), Goldberg and Hachey (1992), and Hardouvelis and Kim (1995). Generally, these papers conclude that margins have served their purpose in protecting the exchanges and FCMs against customer defaults. However, higher margin requirements also decrease market liquidity and increase the cost of hedging. There is disagreement over whether higher margins reduce market volatility.

[11]There are as many rates of return as there are sign changes in the series of cash flows. Some of the rates of return can be negative (or imaginary), and it will not be known which rate is correct, or whether a high or low rate of return will be desired.

[12]Some sources define **basis** as the futures price minus the cash price; therefore, be certain that you and anyone you are conversing with agree on the definition. For example, in talking about index futures, the basis is defined as the futures price minus the cash price. A problem also arises in defining the cash price. Consistency must be maintained for both the quality and location of the spot good. There will be a different basis for each quality and location of the good. For instance, crude oil can have different grades, each with a different price, hence a different basis. Moreover, crude oil of the same grade can sell for different prices at different locations, reflecting relative supply and demand at those places, as well as the transportation costs of moving oil from one place to the other.

[13]Actually, as a result of transactions costs, the basis can be slightly different from zero on the delivery date.

[14]The act of buying the present value of the number of futures contracts is called ***tailing***. The difference between 1.0 and the present value of 1.0 is called the tail. In this example, the tail at time 0 equals $1.0-0.7513148=0.2486852$. This concept will be covered again in the next chapter on hedging with futures contracts.

[15]corr $(\Delta F, \Delta r)$ is the correlation of changes in futures prices with the changes in interest rates. If corr $(\Delta F, \Delta r)>0$, then when futures prices rise, interest rates will also tend to rise. If corr$(\Delta F, \Delta r)<0$, then when futures prices rise, interest rates will tend to fall.

[16]See Fink and Feduniak (1988), Chapter 6, and the Chicago Board of Trade's *Commodity Trading Manual* (1994), Chapter 7, for more discussion on regulation of futures trading. The *Journal of Futures Markets* has published some issues with many articles analyzing futures regulation, such as the Summer 1981 and Fall 1984 issues. Also, other issues contain a bibliography listing the latest published research in futures regulation, such as the April 1995 and April 1996 issues.

[17]Computing the value of the bonds underlying the Treasury bond futures contract is actually more complicated than this. See Chapter 9 for more thorough discussion of this complex contract.

[18]Access the CMEs website to find out the exact delivery dates. Contract specifications can be viewed at www.cme.com/clearing/spex/XMLReports/intrRateGroup.htm. Important dates for the T-bill contract can be viewed at www.cme.com/clearing/listings/tbfut.htm.

[19]Brennan (1986) hypothesizes that price limits serve as a substitute for margin requirements by ensuring that losses on futures contracts will be paid. In other words, margin requirements are smaller because of daily price limits.

[20]Most futures trading occurs in pits at the exchanges. A pit is a roughly circular, or polygonal, depression in the floor with steps leading down to a central desk. Traders stand on the steps of the pit facing each other, and shout out and signal (using their hands) trades they wish to make. Other traders can accept any bid or offer, or shout out and signal alternative terms to any proposed trade. This process is called an **open outcry auction**.

[21]The word "should" is used deliberately. Two traders in the pit can agree on a price for a trade. But if *your* representative is not quick, then in a fast market, he may not always get to participate in the next trade after the specified price is touched, or the best possible price.

[22]Stop orders are also used by technical traders trade to initiate new positions based on a series of previous moves. For example, the current futures price of gold might be $276/oz. A technical trader might believe that if it rises above $280/oz. gold will be in a "bullish formation" and will likely go higher. Therefore, he might place a stop-buy order at 280.20. Note that, the specified buy price is above the current futures price.

[23]Several books exist on how to use fundamental and technical analysis when trading futures. See, for example, Pring (1991) and Schwager (1995).

[24]Silber (1984) provides a fascinating study of scalpers' trading activities. Among other results, Silber found that one typical scalper's average holding time of a position was less than 2 minutes and that money was usually lost on a trade if it was held for more than 3 minutes. Also, 48% of the typical scalper's trades were profitable, 22% led to losses, and 30% provided neither. The mean profit per contract was $10.56, but the scalper also traded about 68 round-trip trades per day; this translates to an expected annual profit of about $179,000.

[25]A "new basis" here means a new initial trade price for tax purposes.

PROBLEMS

6.1 Refer to Figure 6.4, and answer the following questions.

- **a.** What is the value of the gasoline underlying the August gasoline contract?
- **b.** If you were short one August gasoline contract, what is the mark-to-market cash flow on the day shown in Figure 6.4?
- **c.** If you were lucky enough to have bought one October gasoline contract at its lifetime low, and sold it at its lifetime high, what profit would you have realized?

d. If the spot price of gasoline on July 28, 1999, was $0.635/gal, what is the basis of the August futures contract?

e. Is the gasoline futures contract normal or inverted?

f. Why won't firms sell their gasoline on the spot market at $0.635/gal, and buy February futures at the much lower price shown? Isn't this an arbitrage opportunity?

6.2 Suppose you bought one gold futures contract for August delivery at its April 2 settlement price of $279/oz. Assume that both the last trading date and the delivery date are August 27. Assume that your borrowing and lending rates are 8% per year, and that you borrow to meet any mark-to-market cash outflows and lend any mark to market losses.

a. If the gold futures price remains unchanged until August 27, then falls to $250/oz. on that day, what is your profit or loss? (Be sure to *state* whether the result is a profit or loss.)

b. If, instead, the gold futures price falls to $250/oz. on April 3, and stays there until the delivery date, what is your profit or loss?

c. If, instead, the gold futures price rises to $420/oz. on April 3, stays there, and then falls to $250/oz. on August 27, then what is your profit or loss?

d. Given that money has time value, which of the foregoing price scenarios is most attractive: a, b, or c?

6.3 Discuss how futures contracts differ from forward contracts.

6.4 Find the initial and maintenance margin requirements for S&P 500 futures contracts in Table 6.1. Using the March 2001 contract settlement price shown in Figure 6.5, at what futures price will there be a margin call for an individual who goes long one contract?

6.5 If you were short one Canadian dollar futures contract for December delivery, and the *Wall Street Journal* reported the following price information, what would be the mark-to-market cash flow consequences for you?

Month	Open	High	Low	Settle	Change
Dec	.6321	.6340	.6285	.6313	+.0045

6.6 Suppose that you go long one December Treasury bond futures contract at its lowest price for that day, given the following price data. The initial margin requirement is $2700, and the maintenance margin requirement is $2000. At what futures price would you receive a margin call?

Month	Open	High	Low	Settle	Change
Dec	115–02	115–18	114–27	115–03	–7

6.7 Define basis, and compute the (closing) basis for the September S&P 500 futures contract, using the price information in Figure 6.5.

6.8 Suppose you went long one March yen currency futures contract at the settle price shown in Figure 6.5. On the delivery date, the settle price is 0.8500, and delivery takes place. Therefore, to satisfy the terms of the futures contract; must you deliver yen or take delivery of them? How many dollars (the invoice price) will you actually be paid for the yen on the delivery date? What was your mark-to-market profit or loss?

6.9 The crude oil futures contract covers 1000 bbl of oil. One tick is one cent/bbl, and this equals $10/contract. Assume that the initial margin is $3000 and that maintenance margin is $2200. Suppose you buy one crude oil futures contract at 10 A.M, on November 5, when the futures price is $30.45.

The following table contains the subsequent settlement prices:

Day	Settlement Price
11/5	$30.68
11/6	$31.02
11/7	$30.74
11/8	$30.00
11/9	$29.64
11/12	$29.19
11/13	$29.84
11/14	$29.98
11/15	$29.45
11/16	$28.86
11/19	$28.44
11/20	$28.94

You offset your trade on November 20 at 1 P.M when the futures price is 28.81.

a. Prepare a table of daily resettlement cash flows such as in Table 6.2. Be sure to note when maintenance margin calls are received.

b. Suppose instead that you used a bank letter of credit to satisfy the initial margin requirement. What are your variation margin cash flows?

6.10 Explain why the value of a futures contract is zero after it has been marked to market.

6.11 What features make credit risk for a futures contract less than the credit risk contained in a forward contract?

6.12 A corporation plans on borrowing money by issuing bonds next week. Would this firm do a long hedge or a short hedge? Explain your answer.

6.13 Explain what is meant by the following statement: The difference between the initial futures price and the actual cash amount that is paid upon delivery is equal to the sum of the variation margin cash flows.

6.14 A trader notes that the three-month Euroswiss interest rate is 45 basis points higher than three-month Euribor (the interest rate on euros). He investigates, and finds that this had never happened in the preceding five years; usually, Euribor actually exceeds the Euroswiss interest rate. How can the trader use Euroswiss and Euribor futures to speculate that the interest rate anomaly will correct itself? You should refer to actual price data in the financial press to help understand the question.

6.15 Refer to Figure 6.5. Where are three-month interest rates probably the highest: in the United States, the United Kingdom, Japan, the European Union (issuer of euros), or Canada? In which country are they likely the lowest? Why?

6.16 Refer to Figure 6.5. What is the size of one tick (U.S $0.0001/Can. $) for the Canadian dollar futures contract?

6.17 Obtain a recent *Wall Street Journal.* Study the price changes for the preceding day for all the currency futures. For which contract did a short make the greatest mark-to-market profit (or smallest loss)? For which contract did the long make the greatest profit (or smallest loss)?

6.18 Explain how volume can be high, yet open interest can either decline or increase.

6.19 Observe the following price data for feeder cattle:

Delivery Month	Settlement Price
September 2001	$79.50
October	$80.02
November	$81.25

January 2002	\$81.62
March	\$80.72
April	\$80.67

Why wouldn't anyone who owns the actual livestock sell futures for delivery in January 2002 and buy futures for delivery in April 2002? After all, this locks in a selling price in January of \$0.8162/lb, and then the "arbitrageur" will have locked in a buying price in April of only \$0.8067/lb. The existing livestock can be sold off for a price that is higher than the price that would be paid in April to replenish the inventory.

6.20 Look at a recent *Wall Street Journal.* Record the spot price of crude oil (Nymex crude: it's shown on page C1 every day) and the futures price for delivery six months later. Assume that the delivery day for the futures contract is the first of the month. Assume no convenience value, and use the FinancialCAD function aaCDF_repo to compute the repo rate. Verify your result, using the annualized value of $(F-S)/S$ = implied repo rate.

CHAPTER 7

Risk Management with Futures Contracts

7.1 Introduction

Because futures are so similar to forwards, most of the concepts that were covered in Chapter 4, Using Forward Contracts to Manage Risk, also apply when one is hedging with futures contracts. Thus, you should review Chapter 4 before reading this chapter. Often, transactions costs, liquidity, accounting rules, and basis risk will determine which contract is preferred.

In this chapter, we will discuss traditional **hedging theory**, which applies to the use of futures to manage price risk. Liquidity risk and basis risk are important considerations when one is comparing futures and forwards for risk management. Also, forward contracts entail greater credit risk, which is the risk that your counterparty will not abide by the terms of the forward contract. In contrast, futures clearing houses and the mark-to-market process essentially eliminate credit risk when futures are used. Using futures to manage price risk also introduces the following risks:

1. Because futures contracts are standardized, the underlying asset, the delivery location, the quantity, and the delivery date may all differ from the asset that is being hedged. This risk is called **basis risk**.
2. If the underlying asset of the futures contract is sufficiently different from the asset being hedged, it is important to determine the degree to which price changes of the two assets are correlated.
3. If the date of the hedging horizon lies beyond the date of the most nearby futures contract, then the choice the correct delivery date must be made. Frequently, hedges must be **rolled forward** by offsetting the position of nearby contracts and entering into a position in contracts with more distant delivery dates.
4. Because futures are marked to market daily, futures hedges must be **tailed**.

In this chapter, these and other aspects of hedging with futures contracts will be discussed.

Futures contracts enable market participants to alter risks they face that are caused by adverse, unexpected price changes. One of the main reasons cited for the existence of futures markets is that they are a low cost, effective way to transfer price risk. It is no accident that interest rate futures markets did not evolve until the 1970s. Before then, unhedged long or short positions in debt instruments were considerably less risky because their prices rarely changed. Figure 7.1 presents interest (discount) rates on three-month Treasury bills since 1950, and Figure 7.2 is a graph depicting yields available on long-term AAA-rated corporate bonds since 1919. Both graphs show that interest rates fluctuated relatively little during the 1940s, 1950s, and early 1960s. When interest rates became volatile in the late 1960s and 1970s, the demand for ways to hedge against this volatility increased, and the interest rate futures market came into existence. The increase in interest rate risk that began in the late 1960s is further illustrated by Figure 7.3, which graphs the volatility (rolling 60-month standard deviation of monthly interest rates) of Treasury bills and AAA-rated corporates since December 1954. Before the late 1960s, the standard deviation of monthly rates was never above 1%. Volatility did not return to those levels until the late 1990s.[1]

Futures contracts are used to manage risk by taking a futures position that is the opposite of the existing or anticipated cash position. In other words, a hedger sells futures against a long position in the cash asset or buys futures against a short position in the cash asset.

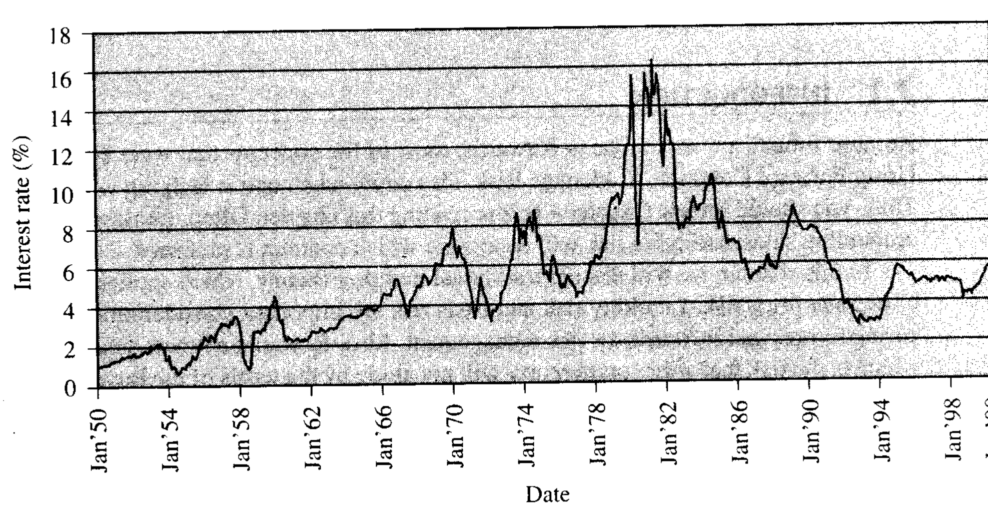

Figure 7.1 Discount rates on newly issued three-month Treasury bills, January 1950–June 2000. (From: www.federalreserve.gov/releases/H15/data.htm.)

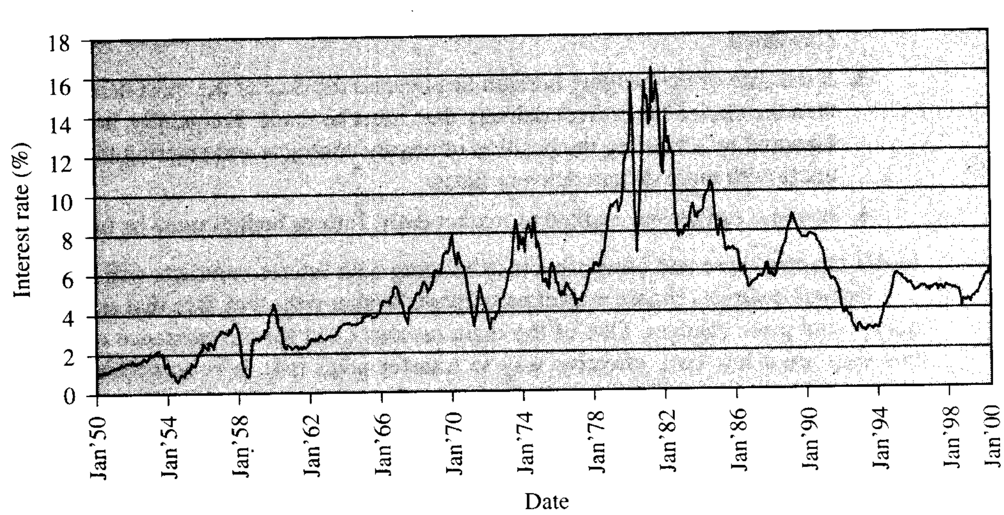

Figure 7.2 Yields to maturity on AAA-rated corporate bonds, January 1919–March 2001. (From: www.federalreserve.gov/releases/H15/data.htm.)

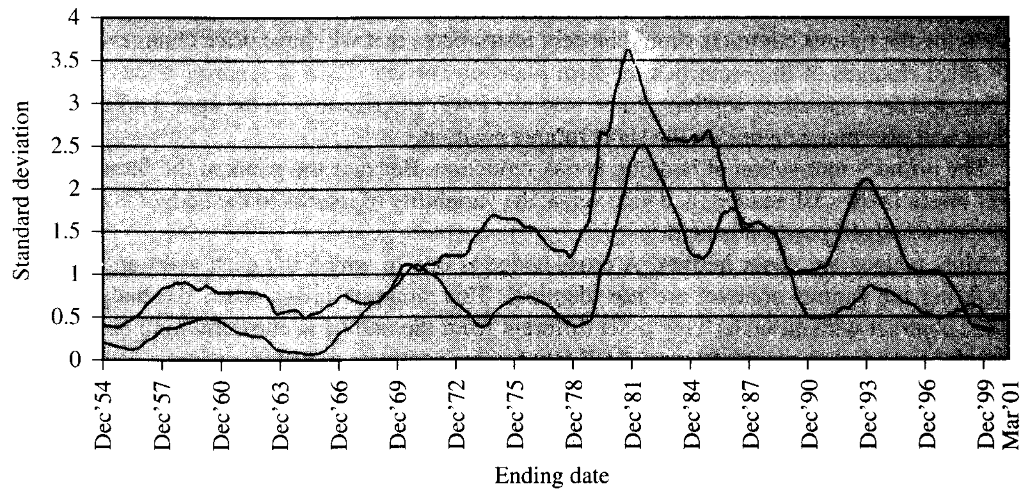

Figure 7.3 Rolling volatility (standard deviation) of three-month discount rates on bills (upper curve) and Treasury AAA-rated corporate bond yields (lower curve), December 1959–March 2001. (From: www.federalreserve.gov/releases/H15/data.htm.)

Futures hedges are often characterized as either long hedges or short hedges. In a **long hedge**, one buys futures contracts. The hedger either is currently short the cash good or has a future commitment to buy the good at the spot price that will exist at a later date (the price is a random variable). In either case, the long hedger faces the risk that prices will rise. Because the long hedger has a long futures position and a short cash position, any subsequent price rise should lead to a profit in the futures market and a loss in the cash market. The hedger must also be aware that prices may fall, in which case a profit will be earned on the spot position, while a loss will be sustained in the futures market. Note that the success of this strategy depends on the linkage between price changes in the two markets; that is, the hedger must be reasonably sure that the changes in the value of his cash position and changes in the futures price will be correlated.

A long hedge might be used by a mutual fund or pension fund money manager who is anticipating the receipt of a large sum of money to invest. The manager expects asset prices to rise between today and the day that the cash will be received. Thus, he should buy futures contracts calling for delivery of an asset (e.g., a stock index or Treasury bond) that is similar to the asset he intends to buy upon receiving the cash. If prices rise, more will have to be paid for the cash assets (a loss in the spot market), but a profit will have been earned on the futures contracts. Long hedges such as this are called **anticipatory hedges**, because they are done in anticipation of a subsequent long position.

In a **short hedge**, futures contracts are sold. Here, the hedger fears that prices will fall, because if they do, losses will be sustained on the spot position. Typically, the short hedger either is currently long the cash good or has a commitment to sell it in the future at an unknown price. With the hedge in place, if prices do indeed decline, losses will occur on the cash position, but profits should be earned on the short futures position.

A firm that is planning on issuing new debt securities (corporate bonds, or commercial paper) in the near future might use a short hedge. It fears that between today and the issuance day, security prices will decline (interest rates will rise). Thus, the firm should sell futures. The asset underlying the futures contracts should be debt instruments that will have price changes correlated with price changes of the securities the firm plans on issuing. Then if security prices do indeed decline, the new securities will have to be sold at a lower price (a loss in the spot market), but the hedger will also realize gains on the short futures position.[2]

The primary motivation of hedging is risk reduction. Because the gains in the futures market offset losses in the cash market, and vice versa, the variability of returns to the hedger is lower than when an unhedged position is held.

Most hedges are **cross hedges**. A cross hedge is one in which the cash asset and the asset underlying the futures contract are not identical. This situation arises when the hedger uses a futures contract with an underlying asset different from the one he is currently long or short. For example, consider a hedger who uses S&P 500 futures contracts to hedge against a decline in the value of his portfolio, which is different from the 500 component stocks in the S&P 500. Or consider an oil refiner who anticipates the purchase of crude oil in the near future. The firm can hedge against possible price increases by going long crude oil futures. This will be a cross hedge if the quality of crude oil the refiner will likely purchase is not the same as the quality of crude oil underlying the futures contract. It will also be considered a cross hedge if the location of the refinery differs from the location at which the crude oil futures contract is being priced, since the price of oil will be different at the two locations. When the price or value of the asset being hedged differs from the price or value of the asset underlying the futures contract, we say that **basis risk** exists.

An investor who uses T-bill futures contracts to hedge against undesired price changes in any debt instrument other than a 90-day T-bill also has a cross hedge. In fact, the only hedging situation that is not a cross hedge with T-bill futures contracts arises when one is anticipating the purchase or sale of 90-day T-bills on the contract's delivery date. If you are currently long $1 million in T-bills that have 90 days to maturity today and hedge with the T-bill futures contract, you are cross hedging, because the futures contract prices 90 day T-bills it *on the delivery date*, not today. Your T-bills may have 90 days to maturity today, but they will have 89 days to maturity tomorrow, 88 days to maturity on the day after that, and still fewer days to maturity on the T-bill futures contract's delivery date. In general, differences in coupon, maturity, or type of debt instrument will create cross hedges when interest rate futures contracts are used.

Because of basis risk, the correlation between the price movements of the futures contract and the cash asset being hedged becomes important. The hedger must be confident that price changes of the spot asset and the futures contract will move together. **Basis** refers to the difference between the price of a cash asset and the futures price[3]:

$$\text{basis} = \text{cash price} - \text{futures price}$$

The relevant cash price differs for different individuals. For a hedger, what is important is the difference between the price of the cash asset that he is long or short (as opposed to the deliverable asset) and the futures price. For example, if a New Jersey refinery is planning to purchase Mexican oil and pay the New Jersey crude oil price, then the basis risk of concern is the difference between the New Jersey crude price and the NYMEX futures price. But the underlying asset of NYMEX crude oil futures contracts is West Texas Intermediate crude oil, priced in Cushing, Oklahoma. For this hedger, the cash asset, defined in terms of the oil's quality and its physical location, differs in quality and location from the crude oil that determines the futures price.

In contrast, when speculators and arbitrageurs discuss basis, they use the price of the deliverable asset underlying the contract. To illustrate, consider the case of the S&P 500 futures contract: a hedger would be concerned with the difference in the value of the stock portfolio being hedged and the futures price of the S&P 500 futures contract. In contrast, an arbitrageur would be concerned with how the futures price deviates from the value-weighted portfolio of 500 stocks that compose the spot S&P 500 index.

Finally, for interest rate futures, basis is sometimes defined as the difference between the forward price and the futures price, or between the forward yield implicit in the today's term structure and the futures yield. The determination of forward interest rates implicit in the spot prices of debt instruments was discussed in Chapter 5. Recall that buying a debt security of one maturity in the spot market and simultaneously selling a debt security of another maturity in the spot market creates a synthetic forward instrument and determines a forward price (and a forward interest rate).

A hedger exchanges price risk for basis risk. To understand this statement, consider an individual who is currently long or short one unit of a cash asset. The current spot price of that asset is S_0. The risk is that the price of the cash asset will change to $\tilde{S}_1$. In other words, the risk faced by an unhedged investor is

$$\tilde{S}_1 - S_0 = \Delta\tilde{S}$$

The hedger should use a futures contract on one unit of an underlying asset that will move in tandem with the cash asset being hedged. Then the risk equals the change in the price of the cash asset minus the change in the futures price:

$$(\tilde{S}_1 - S_0) - (\tilde{F}_1 - F_0)$$

The minus sign used between the two terms in parentheses accounts for the fact that the hedger takes a position in the futures market that is the opposite of the one that exists in the spot market. Rearrange the foregoing statement of risk, we write

$$(\tilde{S}_1 - \tilde{F}_1) - (S_0 - F_0)$$

or

$$\tilde{\text{basis}}_1 - \text{basis}_0$$

Today's basis, basis_0, is known. The basis at time 1, $\tilde{\text{basis}}_1$ is generally a random variable.[4] That is, as of time 0, the basis that will exist at time 1 is usually unknown.

An unhedged individual faces price risk: the risk that the price of the cash asset will change. A hedged investor faces basis risk: the risk that the basis will change. Upon combining the definition of basis, $S-F$, and the theoretical cost-of-carry model, $F=S+CC-CR$ we conclude that basis should theoretically equal carry returns minus carry costs. This is also frequently called the *net cost of carry*:

$$\text{basis} = CR - CC$$

Because of convergence, some of the change in basis is predictable. That is, the basis is known to be zero on the delivery date if the cash asset exactly matches the underlying asset of the futures contract.

If the basis is guaranteed to remain unchanged, or if it is perfectly predictable at the end of the hedging horizon, the investor can create a perfect hedge. Since, however, the basis on the day the

EXAMPLE 7.1 Suppose $S = 100$ and $F = 105$. The basis is -5. Because of convergence, the hedger knows that on the delivery date, the basis will be zero.[5] This situation favors the short hedger who is selling the spot good forward at a forward price that exceeds the spot price. If the spot remains unchanged at 100, the futures price will decline. Even if the spot price rises by 10 points to 110, the futures price will rise only 5 points. An aggressive short hedger might sell more than one futures contract to hedge one unit of the spot position, even if the hedge is expected to be of short duration, and even if the risk minimizing hedge ratio calls for only one futures contract to be sold. Further impetus for speculating on the basis occurs when the futures contract is believed to be mispriced. If the hedger in this example, believed that the basis should theoretically be -4 or -3, then the futures price of 105 is too high, and an aggressive hedger would sell more than the theoretical risk minimizing number of futures contracts.

hedge is lifted is rarely known with certainty, perfect futures hedges are rare in practice. To minimize basis risk, price changes of the cash asset and the futures price must be highly correlated. The higher the correlation between the price changes of the cash asset and futures contract, the lower the level of basis risk. As the asset underlying the futures contract becomes more like the cash asset, the correlation between the two approaches 1.0, and basis risk is reduced.

Some aggressive hedgers will "speculate on the basis." A hedger must decide on the number of futures contract to trade with a view to reducing the overall risk exposure as much as possible. If, however, a hedger believes that the basis will change in some predictable way, this hedger may try to profit by trading fewer contracts than would normally be the case, or additional contracts than normal.

7.2 Some Special Considerations in Hedging with Futures

The most important decision when one is hedging—whether to go long or short futures—requires proper identification of the direction of risk exposure. Determining the proper number of contracts to trade is also very important and will be discussed in Section 7.3. In addition, the decision to hedge with futures requires both the choice of the proper underlying asset as well as the proper delivery month. Choosing the correct underlying asset or assets can be difficult. Because futures are standardized, a contract that precisely matches the underlying asset being hedged may not be traded on any exchange. For example, Treasury bill, Eurodollar, and/or Treasury note futures might be used to hedge a portfolio of money market instruments and short-term debt securities that has an average time to maturity of one year.

Selecting the delivery month can also require analysis. For example, suppose you are expecting a cash flow on July 20, and futures contracts with delivery dates only in June and September exist. You can (a) use June futures and bear price risk between the delivery date and July 20, (b) use June futures today, offset the June contract just before delivery, and then (in June) use September futures to hedge, or (c) use September futures today and bear the basis risk that exists when you offset the September futures position on July 20.

EXAMPLE 7.2 Suppose that in early May, a firm prepares its cash budget for the remainder of the year and concludes that it will experience a cash shortage of about $5 million into early next year. To cover its cash needs, the firm expects to issue short-term debt securities every three months. In a strip hedge, it would sell futures contracts with different delivery months. For example, today (May 12), it might sell five June, five September, and five December Eurodollar futures contracts. Each group of five futures contracts is offset just prior to its delivery date. On the other hand, a rolling hedge would instead have the firm sell 15 June contracts on May 12. Just before the June delivery date, the hedge is rolled over by offsetting the 15 June contracts, and simultaneously selling 10 September contracts. Just before the September delivery date, the firm offsets the 10 September contracts and sells 5 December contracts.

Sometimes there are several hedging horizon dates. For example, a jeweler might anticipate the purchase of gold every three months over the next year or two. These situations require the risk manager to choose between a strip hedge and a rolling hedge (sometimes called a stacking hedge). A **strip hedge** requires the use of contracts with different delivery dates. A **stacking hedge** requires using contracts with only one delivery date, usually the nearest one. As that delivery date nears, the hedger rolls out of the expiring contracts (they are offset) and moves into contracts with more distant delivery dates.

Deciding whether to use a strip hedge or a rolling hedge will depend on several factors. First, the hedger must consider the liquidity of the nearby contract relative to the liquidity of contracts with more distant delivery dates. Sometimes the hedging horizon lies beyond the latest date of any futures contract, in which case the hedge must be rolled over at least once. The nearby contract will usually be more liquid than contracts with distant delivery dates, and therefore will have a narrower bid–asked spread. Thus, when liquidity is a factor, a rolling hedge is usually appropriate, all else equal.

On the other hand, transactions costs will usually be lower when one is employing a strip hedge. In Example 7.2, the rolling hedge requires trading two times as many futures contracts (30) as the strip hedge. Another factor in deciding between the two methods is relative mispricing: Is the contract with the more distant delivery more or less overpriced than the nearby contract? Finally, the basis risk for a rolling hedge is usually greater than the basis risk existing in a strip hedge.

A more concrete example of the strip hedge/rolling hedge decision will be presented in Chapter 10.

It is important to understand the difference between microhedging and macrohedging. A microhedge is one that protects an individual transaction. The concept of macrohedging requires that the firm examine its *overall* exposure to risk factors. In other words, one division of a company may be exposed to the risk that the $/¥ exchange rate will rise, while another division may be exposed to a declining $/¥ rate. Overall, the firm might be hedged, and there is clearly no need for microhedging. Firms should definitely examine their overall risk exposure before deciding whether hedging is needed.

In addition to these decisions, the hedger must determine the optimal number of futures contracts to trade. The next section discusses this complex decision.

7.3 The Hedge Ratio

This section describes two different ways of determining the proper number of futures contracts to buy or sell when one is hedging. Along the way, we will also discuss the factors that dictate the choice of which futures contract to employ as part of the hedge. Note that the process of selecting which futures contract and the number of futures contracts to trade is frequently described as an "art." There is no substitute for gathering as much information as possible and carefully analyzing the data to establish a predicted relationship between the price of the cash good being hedged and a futures price. Naïve use of any one approach without careful thought can lead to costly errors. In both approaches we discuss, the **hedge ratio** is defined to be the ratio between the number of futures contracts (each on one unit of an underlying asset) required to hedge one unit of a cash asset that must be hedged.

For example, if it is determined that 1.2 Treasury note futures contracts are needed to hedge each Treasury note, then the hedge ratio is 1.2. If 0.95 crude oil futures contracts (each on one barrel of crude oil) must be sold to hedge the future production of one barrel of crude, then the hedge ratio is 0.95. Our task is arriving at an approach to determining the hedge ratio.

7.3.1 The Portfolio Approach to a Risk-Minimizing Hedge[6]

Here, we assume that the hedger is interested in risk minimization. Risk is defined to be the variance of portfolio value changes. Price changes are random variables. The current price of the cash asset S_0 and the current price of the futures contract under consideration for hedging purposes F_0 are known. The prices of each at the termination of the hedging horizon, $\tilde{S}_1$ and $\tilde{F}_1$ at time 1, are not known.

Assume that the hedger is long one unit of the cash asset. For ease of interpretation, "one unit" should be defined as the unit of the futures contract (e.g., one barrel of crude oil). Note, however, that the underlying asset of the futures contract may not be exactly the same as the asset being hedged. The current price, or value, of the cash position is S_0. The gain or loss on one unit of the cash position is $1(\tilde{S}_1 - S_0) = 1\Delta\tilde{S}$. The risk of the unhedged position is $\text{var}(1\Delta\tilde{S})$, which equals $1^2\text{var}(\Delta\tilde{S})$, which is the variance of $\Delta\tilde{S}$.[7]

Now, suppose the hedger sells h futures contracts to hedge this position. The gain or loss on the portfolio is

$$1(\tilde{S}_1 - S_0) - h(\tilde{F}_1 - F_0)$$

and the risk of the portfolio is

$$\begin{aligned}
&\text{var}[1(\tilde{S}_1 - S_0) - h(\tilde{F}_1 - F_0)] \\
&= 1^2\,\text{var}(\Delta\tilde{S}) + h^2\,\text{var}(\Delta\tilde{F}) - 2(1)(h)\text{cov}(\Delta\tilde{S}, \Delta\tilde{F}) \\
&= \text{var}(\Delta\tilde{S}) + h^2\,\text{var}(\Delta\tilde{F}) - 2h\sigma(\Delta\tilde{S})\sigma(\Delta\tilde{F})\text{corr}(\Delta\tilde{S}, \Delta\tilde{F}) \qquad (7.1)
\end{aligned}$$

This last result is obtained because $\text{var}(a\tilde{X} - b\tilde{Y}) = a^2\,\text{var}(\tilde{X}) + b^2\,\text{var}(\tilde{Y}) - 2ab\,\text{cov}(\tilde{X}, \tilde{Y})$, where a and b are constants, and $\tilde{X}$ and $\tilde{Y}$ are two random variables. When applying this property of random variables to Equation (7.1), just set $a = 1$ and $b = h$.[8] Finally, also note that:

$$\text{corr}(\Delta\tilde{S}, \Delta\tilde{F}) = \frac{\text{cov}(\Delta\tilde{S}, \Delta\tilde{F})}{\sigma(\Delta\tilde{S})\sigma(\Delta\tilde{F})}$$

To minimize risk, take the first derivative of Equation (7.1) with respect to h:

$$\frac{d[\text{risk}(h)]}{dh} = 0$$

The solution to this equation is

$$h^* = \frac{\text{cov}(\Delta\tilde{S}, \Delta\tilde{F})}{\text{var}(\Delta\tilde{F})} = \frac{\sigma(\Delta\tilde{S})\text{corr}(\Delta\tilde{S}, \Delta\tilde{F})}{\sigma(\Delta\tilde{F})}$$

Now, it is also true that if you ran the following regression model using historical price change data:

$$\Delta S = a + b\Delta F \tag{7.2}$$

the estimated slope coefficient would be[9]

$$b = \frac{\text{cov}(\Delta\tilde{S}, \Delta\tilde{F})}{\text{var}(\Delta\tilde{F})} = h^*$$

In other words, to find the risk-minimizing hedge ratio h^*, you can use historical price data to run a regression like Equation (7.2). The dependent variable is the change in the spot price (or change in the value of the spot position), and the independent variable is the change in the futures price (or change in the value of the deliverable good underlying the futures contract). The resulting estimated slope coefficient defines how many futures contracts to trade, per unit of the spot position, to minimize risk. It is the hedge ratio. The slope coefficient is interpreted as follows:

$$b = \frac{\text{change in the spot price}}{\text{change in the futures price}}$$

If the results of the regression are to be properly applied, you must assume that the historical relationship of price changes will hold reasonably well in the future If you believe that the past is not an accurate portrayal of the future relationship, you should not use the regression approach; instead, you should use the dollar equivalency method described in the next section.[10]

Figure 7.4 depicts the nature of the regression analysis. Point S is placed at the coordinates $E(\Delta S)$, $\sigma(\Delta S)$ which represent the expected change in the spot price and the standard deviation of the spot price, respectively. The curve illustrates the risk–return combinations of different portfolios of a long position in one unit of the spot good and short positions in futures. In other words, S is a portfolio of 100% in the spot good and no futures (an unhedged long position). As futures are sold, the hedger's position moves down and to the left along the curve. The expected portfolio cash flow decreases, and so does the risk. Eventually, the point noted with an asterisk is reached. At that point, the hedger is long one unit of the cash good and short h^* futures contracts. Risk is minimized. If too many futures are sold, then risk begins to increase, and perhaps even worse, the expected cash flow continues to decline. Eventually, point F is reached; F denotes a situation in which there is no spot position, but futures contracts have been sold. If S is expected to increase in price, then F, which is a *short* position in futures, must be expected to decline in price. Thus, F is below the x axis.

Many hedgers will not wish to minimize risk, nor will they want to be at point S, because it has too much risk. They wish to have a higher expected return than a position like that denoted by the asterisk in Figure 7.4 would provide. Such a hedger has an **indifference curve** U in Figure 7.5, and will want to sell fewer futures contracts than another hedger who is extremely risk averse. An indifference curve shows all risk–return combinations that leave an individual equally satisfied.

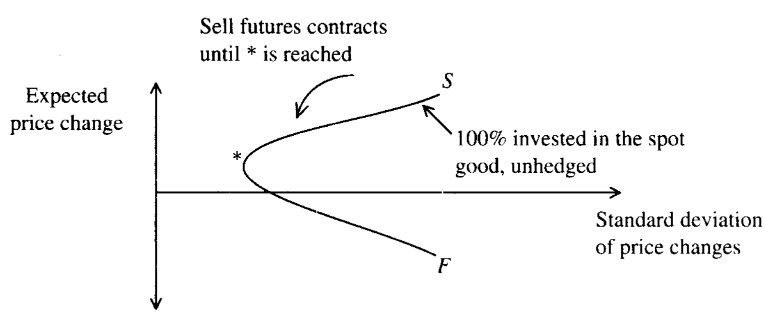

Figure 7.4 How to find the risk-minimizing hedge ratio, as conceptualized by the regression approach. The objective is to find the proper number of futures contracts to sell to reach point *, where the risk of the spot futures porfolio is minimized.

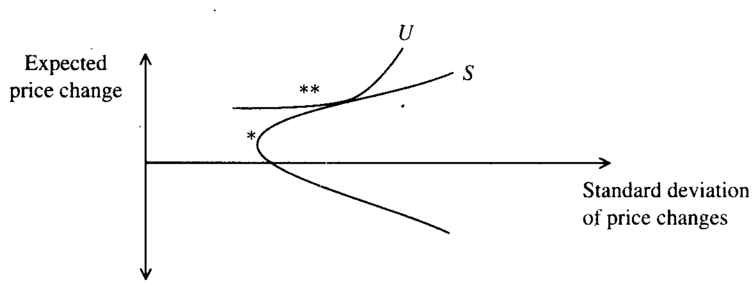

Figure 7.5 Some hedgers may not wish to minimize risk. They will not choose to sell h^* futures contracts to be at point *. For bearing extra risk, they hope to be compensated with additional profit. Given an indifference curve U, such a hedger might sell h^{**} futures contracts and be at point **

For example, an expected price change of $200 with a standard deviation of $20 might be as desirable as an expected price change of $300 with a standard deviation of $35. The curve, U, depicts the trade-offs for one hypothetical hedger. This hedger will be satisfied (utility will be maximized subject to the opportunities available) at the point denoted by two asterisks (**).

EXAMPLE 7.3 An individual is long 100 oz. of gold. The spot price is $300/oz. The expected monthly price change in the spot price of gold is $4/oz., and the standard deviation of monthly spot price changes is estimated to be $15. The futures price for delivery one year hence is $340/oz.[11] The standard deviation of monthly futures price changes is $18. The correlation of monthly changes in the spot price of gold with monthly changes in the futures price of gold is 0.88. Given these data, Table 7.1 illustrates how a diagram like Figure 7.4 is created, and the risk minimizing number of futures contracts is found.

Figure 7.6 is a graph that illustrates the relationship between expected price changes and the standard deviation of price changes. As futures contracts are initially sold, risk declines, until about 0.7 futures contract has been sold. Beyond 0.7 futures contract sold, risk increases. Thus, the risk-minimizing hedge is found to be $h^* = 0.7$.

TABLE 7.1 Risk and Return as a Function of the Number of Futures Contracts Sold

Number of Futures Contracts Sold (h)	Standard Deviation of Cash Flow Changes[1]	Expected Change in Cash Flow[2]
0	$[(15)^2+0-0]^{1/2}=15$	4
0.1	$[(15)^2+h^2(18)^2-2h(15)(18)(0.88)]^{1/2}=13.4432$	3.933
0.2	11.9549	3.867
0.3	10.5641	3.8
0.4	9.3145	3.733
0.5	8.2704	3.667
0.6	7.5180	3.6
0.7	7.1498	3.533
0.8	7.2250	3.467
0.9	7.7305	3.4
1.0	8.5907	3.333
1.1	9.7118	3.267
1.2	11.0145	3.2

[1]The standard deviation of the portfolio's cash flows is found by computing the square root of $\text{var}(\Delta\tilde{S}) + h^2\,\text{var}(\Delta\tilde{F}) - 2h\sigma(\Delta\tilde{S})\sigma(\Delta\tilde{F})\text{corr}(\Delta\tilde{S}, \Delta\tilde{F})$.

[2]The expected monthly change in the portfolio's cash flow is found by the formula $[1\ E[CF(\text{spot})] - hE[CF(\text{futures})]]$. The individual is long one unit of the spot commodity (100 oz.) and short h futures contracts (each covering 100 oz.). $E[CF(\text{spot})] = \$4/\text{month}$, and $E[CF(\text{futures})] = \$0.667/\text{month}$.

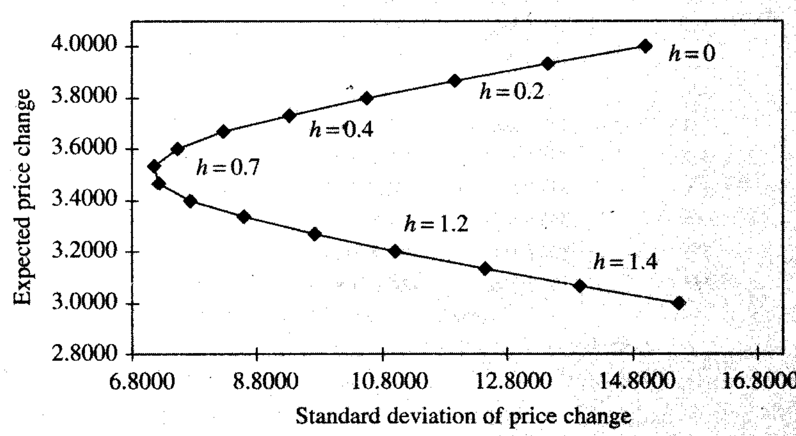

Figure 7.6 Risk and return are a function of the number of futures contracts sold: $h=0$ denotes an unhedged position; $h^*=0.7$ is the risk-minimizing number of futures contracts to sell.

If historical data are available, a hedger can regress historical changes in the monthly spot price of gold on historical changes in the monthly futures price of gold to estimate h^*. In other words, estimate the following model:

$$\Delta S = a + b\Delta F \tag{7.3}$$

where

ΔS = the monthly change in the spot price of 1 oz. of gold

ΔF = the monthly change in the futures price of 1 oz. of gold

The estimated slope coefficient $\hat{b}$ equals h^*.

Consider Table 7.2, which contains hypothetical historical price data. A regression of changes in the spot price (ΔS) on changes in the futures price (ΔF) yields the regression coefficients

TABLE 7.2 Hypothetical Historical Price Data for a Commodity

		Futures Price			Changes, Δ	
Month[1]	**Spot Price**	**Nearby**[2]	**Distant**		**In Spot Price, ΔS**	**In Futures Price, ΔF**
Apr	400	414 Jun	422.2 Sep			
May	410	419.2 Jun	428 Sep		10	5.2
*Jun	406	406.2 Jun	414.3 Sep		−4	−13.0
Jul	388	399.8 Sep	405.4 Dec		−18	−14.5
Aug	394	402.7 Sep	410 Dec		6	2.9
*Sep	377	377.3 Sep	384.8 Dec		−17	−25.4
Oct	397	410.9 Dec	420.7 Mar		20	26.1
Nov	412	418.2 Dec	429 Mar		15	7.3
*Dec	406	405.8 Dec	414 Mar		−6	−12.4
Jan	414	425.5 Mar	434.8 Jun		8	11.5
Feb	412	422.4 Mar	430.8 Jun		−2	−3.1
*Mar	388	387.9 Mar	394.9 Jun		−24	−34.5
Apr	380	391.4 Jun	398.2 Sep		−8	−3.5
May	366	369.9 Jun	375.8 Sep		−14	−21.5
*Jun	371	371.1 Jun	378.5 Sep		5	1.2
Jul	372	378.6 Sep	386.6 Dec		1	0.1
Aug	370	374.9 Sep	382.3 Dec		−2	−3.7
*Sep	366	365.3 Sep	372.7 Dec		−4	−9.6
Oct	379	390.6 Dec	398.8 Mar		13	17.9
Nov	385	396.1 Dec	405.2 Mar		6	5.5
		mean		=	−0.78947	−3.34211
	$\text{var}(\Delta S) = \frac{1}{18}\sum_{j=1}^{19}(\Delta S_j - \overline{\Delta S})^2$			=	142.73099	$\text{var}(\Delta F) = 222.24813$
		std dev$(\Delta S) = [\text{var}(\Delta S)]^{1/2}$		=	11.947008	std dev$(\Delta F) = 14.90799$

[1]An Asterisk indicates a delivery month for the futures.

[2]The futures price change is that of the nearby contract, except in any month following a delivery month. For example, month 4 (July) is after a delivery month. In that month, ΔF is the change in the futures price of the September contract (399.8 − 414.3 = −14.5). Note that the September contract is the distant futures contract in June, but it becomes the nearby contract in July.

$\hat{a} = 1.7525$ and $\hat{b} = 0.761$. The coefficient of determination R^2 is 0.901. Since we argued earlier that the estimated slope coefficient $\hat{b}$ equals

$$\frac{\text{cov}(\Delta S, \Delta F)}{\text{var}(\Delta F)}$$

and this, in turn, was the definition of h^* for a risk-minimizing hedge, we can conclude from this regression that 0.761 futures contract (each on one unit of the underlying asset) should be sold to hedge one unit of the spot position. If the hedger owned 800 units of the underlying asset, and each futures contract covered 100 units, then 6.088 futures contract should be sold. Problem 7.14 at the end of the chapter asks you to use Excel to run this regression model.

In other words, regressing spot price changes on futures price changes yields h^*, which is the risk-minimizing hedge ratio. When hedging a given quantity of an asset, multiply h^* by the number of units of the spot good per the number of units covered by a futures contract. If 800 units are to be hedged, and one futures contract covers 100 units, then the risk-minimizing number of futures contracts to sell is:

$$\begin{array}{l}\text{number of futures contracts} \\ \text{to trade to have a} \\ \text{risk - minimizing hedge}\end{array} = h^* \times \frac{\text{quantity of the cash asset to be hedged}}{\text{quantity of the asset underlying one futures contract}}$$

$$= 0.761\left(\frac{800}{100}\right) = 6.088$$

Sometimes it is helpful to construct a table like Table 7.3 to conceptualize the hedging process. In Table 7.3, we assume that the futures price subsequently changes exactly as predicted by the regression model of spot price changes on futures price changes. The price change in the spot market is $-\$11.60$. ($388.4 - 400 = -11.60$). Since $\hat{b} = 0.761$, the model predicts that the futures price should have declined by \$15.24. ($-11.60/0.761 = -15.24$). Therefore the futures price on the day the hedge is lifted is predicted to be \$424.76. ($440 - 15.24$).

In the example illustrated in Table 7.3, the hedger realizes a loss in the spot market equal to \$9280. But because the inventory of gold was hedged by selling 6.088 gold futures contracts, there was also a profit of \$9278 in the futures market. This was a perfect hedge (except for some rounding error). When the hedge was initiated ("today"), the risk-minimizing hedge ratio was estimated

TABLE 7.3 A Short Hedge

Cash, or Spot, Market	Futures Market
Today	Today
You own 800 oz. of gold. The spot price of gold is \$400/oz. You fear that the price of gold will decline.	Sell 6.088 gold futures contacts at the futures price of \$440 oz.
T days hence	*T* days hence
The spot price of gold has declined to \$388.40/oz.	Buy 6.088 gold futures contracts at the futures price of \$424.76 oz.
Loss: 800(388.4 − 400) = \$9280	Profit: 6.088(\$44,000 − \$42,476) = \$9278

to be 6.088 contracts. This was based on the historical price data presented earlier, where h^* was estimated to be 0.761. This means that if ΔS was \$11.60/oz. then the *predicted* ΔF was \$15.24/oz. (11.60/0.761), which also turned out to be the *actual* ΔF. The hedge thus turned out to be perfect in the sense that the loss in the spot market equaled the gain in the futures market.

In other cases, the hedge might not have been perfect. The change in the futures price might have been greater or smaller than \$15.24/oz. For example, the futures price on the day that the hedge was lifted might have been \$428/oz. Then, the profit on the futures position would have been only \$7305.60, or (6.088)(\$44,000 − \$42,800). The uncertain basis on the day the hedge is lifted illustrates how basis risk affects hedging effectiveness. Basis risk is the risk that the basis ($\tilde{S}_1 - \tilde{F}_1$) will unexpectedly widen or narrow.

Also be aware that these examples ignore the daily resettlement of the futures contracts. In Section 7.4 we will discuss the concept of "tailing." Tailing attempts to account for the fact that profits or losses on futures are realized daily, while the profit or loss on the spot position is not realized until the end of the hedging horizon.

Again, the hedger *must* be confident that a reliable relationship exists between price changes of the spot asset being hedged and price changes of the futures contract. When one is using historical price data to estimate h^*, the reliability of the relationship is typically measured by R^2, which is the **coefficient of determination.** The coefficient R^2 is the square of the correlation coefficient of the two variables in the regression, and it ranges from a low of 0.0 to a high of 1.0. R^2 measures the percentage of the variability of the dependent variable (ΔS) that can be explained by variability of the independent variable (ΔF). When obtained with historical data, it is an anticipated (ex-ante) measure of hedging effectiveness. The higher the R^2, the more effective the hedge *should* be. We say, "should" because the actual hedge (ex-post) can be better or worse than anticipated. Always be cognizant that historical relationships might not persist into the future. Also note that lower R^2 values imply greater basis risk.

When deciding which of two possible contracts (with different underlying assets) to use to hedge a spot position, a useful approach is to run the historical price change regression model, Equation (7.2) and find the R^2 of each. All else equal, the futures contract with the higher R^2 should be used, since, based on historical data, it has the more reliable relationship with the spot commodity. Figure 7.7 illustrates how two futures contracts can obtain the same slope coefficient in a regression, yet have vastly different values of R^2.

Besides R^2, another important variable to consider when deciding which futures contract to employ for hedging purposes is the liquidity of the contract. If one contract has a higher R^2 but lower liquidity, the hedger must evaluate how much liquidity she is willing to give up to obtain a more reliable hedge. Of course, she must also have confidence that the historical relationship that was so very reliable (high R^2) in the past will persist in the future.

The last factor to consider when deciding which futures contracts to use is the relative under- or overpricing of each contract. When doing a long hedge, buy the contract that is cheaper (the one for which the actual F minus the theoretical F is most negative or least positive). For a short hedge, sell the most overpriced futures contract, all else equal. This is the futures for which $F-(S+CC-CR)$ is most positive or least negative.

It is usually advisable to underhedge as your confidence in the future correlation between ΔS and ΔF declines: that is, as the value of R^2 declines, trade fewer contracts than the number called for by h^*. If R^2 is below some arbitrary value, perhaps 0.5, then it is probably not wise to use that particular futures contract to hedge at all. As illustrated in Figures 7.4 through 7.6, there may be a great cost to overhedging in terms of lower expected returns and higher risk.

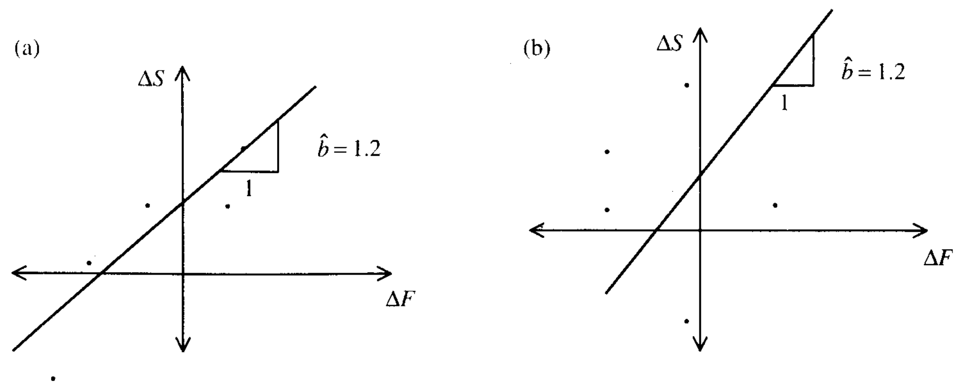

Figure 7.7 The estimated slope coefficient is the same in both regression models: (*a*) a very reliable relationship (i.e., a high R^2) and (*b*) an unreliable relationship (i.e., a very low R^2). Very little of the variation in ΔS is explained by ΔF in (*b*).

In the example developed in connection with Table 7.2 monthly data were used to develop the relationship between spot and futures price changes. An important question to ask is whether monthly data should always be used, or whether other time measurement intervals are adequate or preferred. When one is hedging for only one day, daily price change data should be used in the regression to estimate h^*. Similarly, if the anticipated hedging horizon is only two, three, four, or five days, one should obtain historical spot and futures prices two, three, four, and five days apart, respectively. In other words, for hedging horizons of five days or less, it is recommended that the investor match the anticipated length of the hedge with the interval between historical price data observation points.

Beyond one week, it becomes irrelevant whether weekly, biweekly, or monthly observations are used. But using observations that are *more* than one month apart creates two problems. First, because the estimation error of the slope coefficient (hedge ratio) declines as the number of observations increases, more observations are almost always better than fewer observations. In other words, more observations will increase the accuracy of the estimated slope coefficient in a regression. A more reliable estimate will be obtained from 36 monthly observations than from 12 quarterly observations. Second, if the data go back in time too far, it becomes increasing likely that a different relationship between the spot asset price and the futures price existed at the earlier dates. This also reduces the reliability of the estimated slope coefficient in terms of predicting the future.

On the other hand, observations that are taken only one or a few days apart create another problem that does not exist with weekly or monthly observations. This problem arises from nonsynchronous trading and other "noisy influences" that exist for short measurement intervals. The time at which the futures market closes may not coincide with the time the spot price is measured. Observed prices might be customer sales at the bid price or customer purchases at the asked price. These problems create a less accurate measurement for the relationship between spot and futures prices when they are observed daily.

If your hedging horizon is only one or a few days, you must also recognize that you face greater basis risk. In one day, it is possible that the price changes of the spot good and the futures contract will actually be in *opposite* directions. This is less likely to occur if your hedging interval is a week or longer. Usually, this phenomenon will show up in historical price change regressions

[Equation (7.3)] with a lower estimated slope coefficient and a lower R^2. The moral: be cautious when hedging for only one or two days.

7.3.2 Dollar Equivalency

The goal of a hedge is to avoid losses in the cash market resulting from adverse price changes. Therefore, an alternative method of determining how many futures contracts to trade as part of a hedge is to equate the anticipated loss in the cash market if the spot price changes by some amount to the likely gain in the futures market that will occur concurrently with the spot price change. In other words, define the following:

ΔV_S = change in the value of the spot position

ΔV_F = change in the value of one futures contract

h = number of futures contracts

Then the goal of the hedger is to equate:

$$\Delta V_S = h \Delta V_F$$

Sometimes, spot price data are not available to run a regression. At other times, you might suspect that the historical relationship between spot and futures price changes will not exist in the future. Therefore, you might prefer to use a subjective estimate of the relationship between the change in the value of the spot position and the contemporaneous futures price change. In these instances, employ the dollar equivalency approach to finding h^*. The two approaches can serve as checks on each other, increasing the likelihood that the proper number of futures contracts will be being traded.

EXAMPLE 7.4 An oil producer has 100,000 barrels of oil to transport to a refinery. If the spot price changes by some amount, say \$1/bbl, the resulting change in value of the inventory position, ΔV_S, will be \$100,000. The producer estimates that the contemporaneous change in the futures price of crude oil on the New York Mercantile Exchange will be \$1.02, and thus the change in the value of one futures contract will be \$1020 (one futures contract covers 1000 bbl of a specific grade of oil, priced at a specific location). To neutralize the spot market loss of \$100,000 that would occur if the spot price fell by one dollar, sell $h = \Delta V_S / \Delta V_F = 100{,}000/1020 = 98.04$ futures contracts.

Note that the dollar equivalency approach likely produced the same result as the regression approach. Had the producer used historical price data to run the regression model

$$\Delta(\text{spot price of oil}) = a + b\,\Delta(\text{futures price of oil})$$

he might have found $\hat{b} = h^* = 0.9804$ (which says that if the futures price falls by \$1.02/bbl, the spot price should fall by \$1/bbl). Then, multiplying 0.9804 by the number of units of the spot position (100,000) divided by the number of units of oil in one futures contract (1000), we find that, again, the producer should sell 98.04 futures contracts.

As another example, suppose you own a portfolio of corporate bonds and wish to hedge against a rise in interest rates that would cause their value to decline. A regression by means of historical price data might not be desirable for many reasons. For instance, the maturity of the bonds is continuously changing. Consequently, the relationship between spot price changes and futures price changes that existed several years ago will likely not exist today. In addition, some of the bonds may not be publicly traded, which means that no price data are available. Furthermore, the default risk, hence the bond ratings, of some of the bonds might have changed, altering the historical relationship between their price changes and those of Treasury bonds. Finally, the way in which Treasury bond futures prices change as interest rates change is highly dependent on the existing term structure of interest rates.

Therefore, using the dollar equivalency method for determining the appropriate number of futures contracts with which to hedge is sometimes the best alternative. Simply stated, when employing the dollar equivalency approach, the hedger must:

1. Estimate the loss in the spot position that will occur if the spot price changes by some arbitrary amount.
2. Estimate the change in the futures price that will occur if the spot price changes by that arbitrary amount.
3. Compute the change in the value of one futures contract given that change in the futures price.
4. Trade the appropriate number of futures contracts so as to realize a profit in the futures market equal to the spot market loss.

Additional specific examples that employ the dollar equivalency approach will be presented in Chapter 9.

7.4 Tailing the Hedge

In the hedging examples just presented, the impact of marking to market cash flows for the futures contracts was ignored. In reality, a long hedger will experience daily resettlement cash outflows on the futures position if futures prices fall. Although these cash flows should be recouped at the termination of the hedge through higher profits on the spot position, there are two consequences of the interim cash outflows.

1. There is an opportunity cost to any daily resettlement cash *outflows*; they must be financed through borrowing, to avoid losing interest by a reduction in the hedger's earning assets. On the other hand, should prices rise, the long hedger will *receive* mark to market cash *inflows*, and these can be reinvested to earn interest. Basically, daily resettlement inflows are desired because they can be reinvested, while daily resettlement outflows are unwanted because they have an opportunity cost.

2. There is a danger that the cash outflows could accumulate to such a point that the hedger's assets become depleted, and will be unable to obtain additional funding. The hedge must then be liquidated prematurely.[12]

The first consequence can be handled by tailing the hedge. In Section 6.5 (see note 14 in particular), we showed how tailing neutralizes the valuation effects of marking to market of futures contracts. If future interest rates are nonstochastic (known), then tailing effectively transforms the futures contract into a forward contract. The same concept exists in hedging. Tailing requires that you trade only the present value of the number of futures contracts found to be optimal using the principles covered in Section 7.3. A tail is particularly crucial when interest rates are high and the hedging horizon is long.

EXAMPLE 7.5 A jewelry manufacturer who has 2000 oz. of gold in inventory fears that prices will fall over the next 60 days. The current spot price of gold is \$350/oz. and the futures price for a contract that expires 60 days hence is \$359. The hedger knows that the basis of −9 will strengthen toward zero over the hedge period, and this movement of the basis will work in her favor, since she is a short hedger. The spot interest rate is 12%/year. Naively, the manufacturer proceeds to sell 20 futures contracts, each covering 100 oz. of gold, to hedge her inventory. This is an untailed hedge.

If she is wrong, and prices instead rise over the next 60 days, the manufacturer will experience mark-to-market cash outflows on the short futures position. Assume that initial margin is satisfied by using a bank letter of credit. All variation margin cash flow losses must be paid in cash and must be financed by borrowing. Suppose that the futures price remains constant except on the following days:

Day	Days Remaining in Hedge	Futures Price	Additional Margin Required	Interest on Borrowing Variation Margin Until Hedge Termination
3	57	\$365	\$12,000	$\$12{,}000 \times 0.12 \times 57/365 = \$224.88$
18	42	\$370.20	\$10,400	$\$10{,}400 \times 0.12 \times 42/365 = \$143.61$
51	9	\$377	\$13,600	$\$13{,}600 \times 0.12 \times 9/365 = \$40.24$

On the hedge termination date (day 60), the spot price of gold is \$372 and the futures price is \$372 (convergence). The manufacturer lifts the hedge by offsetting the futures contracts, and, ignoring for now the added interest expense on borrowing to make daily resettlement payments, observes the following results:

	Spot Market	Futures Market
day 0	Has 2000 oz. of gold in inventory: spot price = \$350	Sells 20 futures contracts at the futures price of \$359
day 60	Sells 2000 oz. of gold at new spot price of \$372 profit = \$44,000	Offsets futures position by buying 20 contracts at \$372 each loss = \$26,000

The difference between the spot market profit and futures market loss arises because the basis narrowed from −9 on day zero to 0 on day 60. The narrowing in the basis created the net \$18,000 profit on the hedge (20 contracts times 9 points times \$100/point = \$18,000). Recall that the hedger expected the basis to narrow, so this profit may have been entirely expected.

However, the \$26,000 loss in the futures market ignores the interest required to finance the mark-to-market cash outflows that occurred. Put another way, the hedger would have lost \$26,000 had she originally sold a *forward* contract covering 2000 oz. of gold at a forward price of \$359/oz. But because *futures* were sold, we must also account for the interest lost on the interim mark-to-market cash outflows, which totaled

($224.88+ $143.61+$40.24=) $408.73. Because futures were sold, and futures prices subsequently rose, these daily resettlement cash outflows must be financed.

By applying a tail to the original hedge ratio of 20 contracts, the hedger can almost completely neutralize the effects of daily resettlement. Assume that the interest rate is constant. A tail is created by reducing the number of contracts initially sold from 20 to the present value of 20. On the initiation day of the hedge, the present value of 20 is

$$\frac{20}{1+0.12(60/365)}=19.613$$

On every subsequent day, additional (fractions of) futures contracts should be sold, so that the hedger is always short the present value of 20 futures contracts. Thus, one day after the hedge is initiated, the hedger should be short a total of

$$\frac{20}{1+0.12(59/365)}=19.619$$

contracts.

The *size of the tail* is the difference between 20 and the number of contracts actually sold on any day. As time passes, the size of the tail becomes smaller. By the hedge termination date, there are 20 contracts sold, and the tail is zero. Tailing neutralizes the effect of daily resettlement cash flows by making the interest earned on the investment of mark-to-market cash inflows or paid to finance mark-to-market cash outflows equal to the impact of the hedge ratio reduction. The tail works as follows.

Day	**Days Remaining in Hedge**	**Short Position Contracts Sold**	**Futures Price**	**Additional Margin Required**	**Interest on Borrowing Margin Until Hedge Termination**
0	60	19.61311	$359	—	—
3	57	19.63210	$365	($600)(19.6321) =$11,779.26	($11,779.26)(0.12)× (57/365)=$220.74
18	42	19.72760	$370.2	($520)(19.7276) =$10,258.35	($10,258.35)(0.12)× (42/365)=$141.65
51	9	19.94100	$377	($680)(19.9410) =$13,559.88	($13,559.88)(0.12)× (9/365)=$40.12
60	0	20.0	$372	($500)(20.0) =$10,000 (profit)	—

Now, add all the mark-to-market cash flows. There is a net cash outflow of $25,597.49 ($11,779.26+$10,258.35+$13,559.88−$10,000). In addition, the financing charges totaled $402.51 ($220.74+$141.65+$40.12). The total loss on the futures position and the financing charges is $26,000 ($25,597.49+$402.51). The tail has effectively transformed the futures contracts into a forward transaction. Had a forward contract been originally sold, the hedger would have lost $26,000 on the forward contract. By tailing the futures trades, an equal loss is realized on the futures contracts.

The effect of a tail is that the optimal hedge ratio will always be smaller than the one originally computed by means of the hedge ratio models we have discussed. The number of futures contracts to trade is just the *present value* of the risk-minimizing number of futures contracts. Tailing removes the impact of daily resettlement of futures. In Example 7.5, the tail effect is so small that it can likely be ignored (the hedger would have to decide to initially sell either 19 or 20 contracts). But when interest rates are high, and/or it is expected that the hedge will be in effect for a long time, the tail can be significant. Imagine, for example, a one-year hedge when interest rates are 20%/year.

7.5 Managing the Futures Hedge

Hedgers recognize the price risk they face, and they hedge that risk by purchasing or selling the proper number of futures contracts with the appropriate underlying commodity. The hedger first estimates h^*, using either the regression approach or the dollar equivalency approach. This hedge ratio, h^*, should then be multiplied by the number of units of the spot position being hedged relative to the number of units underlying one futures contract. A tail should be applied if necessary. The final task is to monitor, adjust, and evaluate the hedge. Several examples that illustrate the need to monitor environmental conditions are now presented.

The hedger must realize that conditions change over time, and that these changes might warrant altering the hedge, or removing it altogether. For example, risk exposure might change as assets and liabilities change. A jeweler who has hedged his inventory of gold might initially compute some number of futures contracts to sell based on his net gold price risk exposure. If a few days later he unexpectedly liquidates a large portion of his inventory, the number of futures contracts needed to create an effective hedge obviously must be reestimated.

It is also possible that a sophisticated hedger has developed a model that reasonably accurately predicts the future price of the good he has hedged. The signal provided by that model might change, calling for the hedge to be lifted. In fact, most hedgers are selective in nature, rather than continuous. That is, they hedge only when they perceive a substantial risk that adverse price moves are forthcoming.

In another case a bank has initially hedged a loan, but as time passes and the duration of the loan declines, the loan balance may change. Hedges that employ interest rate futures are particularly dynamic and require a great deal of monitoring because the passage of time alters the nature of the asset or liability being hedged.

A hedger's level of risk tolerance might also change over time. On the initiation date of a hedge, it might be felt that business will be strong over the next year, so that risk tolerance is relatively high. Then, at a later date, new information might increase the perceived probability of a business downturn, decreasing the level of risk tolerance, and calling for a more aggressive hedging policy. Similarly, changes in the level of competition a firm faces might dictate changes in its hedging decisions.

Sometimes another futures contract becomes more favorably priced for the hedger. In this case, the initial contract used to hedge should be offset, and the better priced futures employed instead. Additionally, the hedger should always be aware of upcoming delivery dates that will force the hedge to be rolled over. Thus, the hedger should monitor the relative pricing of the nearby and adjacent contracts. With the right information, the rollover trade can be entered as a calendar spread order, thereby reducing commission charges and allowing the hedger to trade when the spread is believed to be favorable.

Finally, all hedges should be evaluated against the hedger's objectives. What was the profit or loss on the futures position, and on the spot position being hedged? If one has been selectively hedging because of an anticipated adverse price change, one must ask whether that expected price change occurred. Do beneficial price changes occur when one is deciding not to hedge? Was the proper hedge ratio used? What was the (ex-post) risk-minimizing hedge ratio, and why was it different from the (ex-ante) h^* that was originally estimated?[13]

Note that many times a loss is realized on the futures position and a gain on the spot position. If you are a selective hedger, and this occurs more than 50% of the time, you should revise your strategy. These results suggest that you are not adept in predicting future spot price changes and/or not skilled at uncovering mispriced futures. You might consider a continuous hedging strategy, or not hedging at all.

7.6 Summary

This chapter provided details specific to hedging decisions using futures contracts. Corporations, banks, institutions, and individuals face price risk; that is, if the price of a good changes, wealth is affected. Futures contracts can be used to manage that risk. By buying futures when a price rise leads to a loss in wealth, and selling futures when price declines are undesired, one can avoid the adverse consequences of price changes.

The hedger should first identify the net exposure to risk. Will a price rise or a price decline hurt? This determines whether futures should be bought or sold. Futures price changes should be highly correlated with price changes of the asset being hedged, and the futures contract should be as liquid as possible. The hedge ratio determines the number of futures contracts that will lead to a risk-minimizing hedge, although some hedgers may not wish to have such a position. The hedge ratio can be estimated using either the regression (portfolio) approach or the dollar equivalency approach. Hedges should be tailed when interest rates are high and/or hedging horizons are long. Finally, hedges should be actively monitored and evaluated.

Chapters 8, 9, and 10 present additional hedging examples dealing with stock index futures and interest rate futures.

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Notes

[1]The data are obtained at the Fed's website (www.federalreserve.gov/Releases/H15/data.htm).

[2]The underlying assets of interest rate futures contracts are securities. Thus, an individual who wishes to profit from a rise in interest rates (a decline in debt instrument prices) will *sell* futures. Be cognizant of the difference between this situation and that of a forward rate agreement (FRA), which is a forward contract on an interest rate (*not a security*), so that an individual who uses a FRA to profit from a rise in interest rates will actually *buy* the FRA.

[3]Note that some market participants define basis as the futures price minus the cash price. Moreover, it is common to define basis as $F-S$ for some contracts (in particular financial futures), and $S-F$ for other contracts.

[4]The basis at time 1 is not random if time 1 is the delivery date of the futures contract, the good being hedged is exactly equivalent to the futures' underlying asset, and the hedged good exists at a delivery location of the futures contract. Then, $\text{basis}_1=0$ because of convergence.

[5]Any time that the basis increases, i.e., it becomes less negative or more positive, it is said to *strengthen*. Basis *weakens* when it becomes less positive or more negative. In this example, basis is expected to strengthen from -5 ("5 under") to zero. Basis (where the spot asset is the asset underlying the futures contract) is guaranteed to be zero on the delivery date (convergence) by the force of arbitrage.

[6]Johnson (1960) and Stein (1961) are widely cited as the seminal works that integrated hedging theory and portfolio theory. Ederington (1979) is the first to apply the model to interest rate hedging.

[7]If $\tilde{X}$ is a random variable and a is a constant, then $\text{var}(a\tilde{X})=a^2\,\text{var}(\tilde{X})$.

[8]Perhaps you have learned some portfolio theory, in which case the following analogy might be useful. Consider an investor who invests $w_1\%$ of his wealth in asset one and $w_2\%$ in asset 2; $w_1+w_2=100\%$. The returns on assets 1 and 2 are random variables denoted $\tilde{R}_1$ and $\tilde{R}_2$. Then the variance of the returns on the portfolio is $\text{var}(w_1\tilde{R}_1+w_2\tilde{R}_2)=w_1^2\,\text{var}(\tilde{R}_1)+w_2^2\,\text{var}(\tilde{R}_2)+2w_1w_2\,\text{cov}(\tilde{R}_1,\tilde{R}_2)$. This is virtually identical to the material presented in the text, except that the hedger is long an asset and *short* a futures contract.

[9]Continuing with the portfolio theory analogy, recall that a stock's beta coefficient is estimated by regressing the stock's returns on the market portfolio's returns:

$$R_i=\alpha+\beta R_m$$

and that the estimated beta, the slope coefficient, is

$$\beta=\frac{\text{cov}(R_i,R_M)}{\text{var}(R_M)}=\frac{\sigma(R_i)corr(R_i,R_M)}{\sigma(R_M)}$$

[10]Cecchetti, Cumby, and Figlewski (1988) cite several problems with using the regression model to estimate a hedge ratio: some hedgers may not want to have a risk-minimizing hedge (cf Figure 7.4 and Figure 7.5), the relationship between ΔS and ΔF will often vary over time; and historical data are frequently not accurate predictors of what is expected to prevail in the future. These authors' method of estimating a hedge ratio, however, is beyond the scope of this text.

[11]Because of convergence, we can calculate the expected monthly futures price change. $S_0=\$300$. Since $E(\Delta S)=\$4/\text{month}$, then $E(S_T)=\$348$ for $T=1$ year hence. $F_0=\$340$. $E(S_T)=E(F_T)=\$348$. Because the futures price is below the expected future spot price, the situation is one of *normal backwardation*. The expected futures price change over the next year is \$8, and the expected monthly futures price change is \$0.667.

[12]Kolb, Gay, and Hunter (1985) examine the nature of this second consequence.

[13]To estimate the ex-post-risk-minimizing hedge ratio, use the *actual* changes in prices during the interval of time that the hedge was in effect to estimate the slope coefficient in the regression model $\Delta S=a+b\,\Delta F$. The ex-ante h^* was originally estimated at the inception of the hedge from historical data prior to time 0. A change in the relationship between futures price changes and price changes of the asset being hedged might explain much of why a hedge did not perform as expected.

PROBLEMS

7.1 A government securities dealer has a large amount of Treasury bills in inventory. Will he suffer a loss in wealth if T-bill prices rise or fall? Will he suffer a loss in wealth if short-term interest rates rise or fall? If futures are used to manage risk, should he be a long hedger or a short hedger? If he decides to use FRAs instead, will he buy or sell them?

7.2 A U.S. corporation will receive millions of euros from a customer next month. Will the U.S. corporation suffer a loss in wealth if the dollar price of euros (the \$/€ exchange rate) rises or falls? Should the firm be a long hedger or a short hedger if futures are used?

7.3 If basis = cash − futures, does a short hedger hope that the basis strengthens (increases) or weakens (decreases) during the time that her hedge is in place?

7.4 What are some alternative ways to hedge besides using futures contracts? What are some factors a hedger should use in deciding which mode of hedging to use?

7.5 Choose a long-term government bond, and use a newspaper, such as the *Wall Street Journal*, to obtain about 24 prices on 24 consecutive trading days. Also record the futures prices for the nearby Treasury bond futures contract on those 24 days. Regress the changes of the bond's price on changes of the futures price to obtain h^* = the estimated slope coefficient. The regression model is $\Delta S = a + b\Delta F$.

7.6 You work for a sheet metal producer. The firm plans on purchasing 100,000 lb of tin next month, and you have been requested to hedge the planned transaction using futures contracts. Will you do a long hedge or a short hedge?

There are no tin futures. However, you have regressed changes in the spot price of tin on changes in silver futures prices and in copper futures prices. Define ΔT, ΔS, and ΔC as the change in prices of tin, silver futures (\$/oz.) and copper futures (\$/lb). Here are your regression results:

$$\Delta T = 0.001 + 0.25\Delta S \quad R^2 = 0.56$$

$$\Delta T = -0.008 + 1.92\Delta C \quad R^2 = 0.72$$

a. Which futures contract appears to be more suitable for your hedging purposes? Why? What other possible information might induce you to switch to the other contract?

b. How many futures contract of the preferable commodity should be used to hedge the planned purchase of 100,000 lb of tin? Note that you must find out how many pounds of copper or ounces of silver underlie each futures contract.

7.7

a. Suppose that you wish to hedge the planned purchase of 20,000 bbl of crude oil. The purchase will take place five months hence. Interest rates are constant at 16%/year. You have estimated the following regression model:

$$\Delta S = 0.003 + 1.109\Delta F$$

where ΔS is the change in the price of the grade of oil that you will be purchasing, and ΔF is the change in the crude oil futures price. Tail your hedge. What is the risk-minimizing number of futures contracts you should trade to hedge your planned purchase? Will you buy or sell the futures?

b. Suppose that when you first instituted the hedge, S = \$21.40/bbl and F = \$19.93/bbl. During the next five months, the futures price changes as follows:

t	Months Until Hedge Will Be Lifted	F
0	5	\$19.93
1	4	\$20.04
3	2	\$20.95
4	1	\$21.09
5	0	$\$22.35 = F_T$

Do not forget that you have tailed your hedge. What is your final profit or loss on the futures contracts? If S_T= \$23.82/bbl on the hedge-lifting date, what is the final effective purchase price for the 20,000 bbl? In other words, consider both the actual price that you paid for the oil and the profit or loss on the futures contracts. Note that $S_T \neq F_T$ because the grade of oil that you are buying is not the same grade that underlies the futures contract.

c. What was the original basis for your grade of oil? What was the final basis? Suppose that S_T= \$24.50/bbl (instead of the \$23.82/bbl that you assumed in part **b**. Now what is the final, effective purchase price for the 20,000 bbl?

d. Discuss why your answers to parts **b** and **c** differed. Be sure to comment on the meaning of "basis risk."

7.8 Your firm plans on issuing 10,000 pure discount (no-coupon) notes with two years to maturity. Each note has a face value of \$1000; that is, at maturity, it will be worth \$1000. The current yield to maturity on two-year notes like these is 10%/year. You believe that if the Eurodollar futures yield changes by 10 basis points, the change in the required rate of return (yield to maturity) on the notes will be 7 basis points. The mark-to-market cash flow is \$25/basis point for Eurodollar futures.

Use the principles of dollar equivalency to compute the proper number of Eurodollar futures contracts to trade in order to hedge the planned issuance of the notes. Will you buy or sell the futures contracts?

7.9 You are a gold producer, and your plans indicate that you will have 10,000 oz. of gold to sell one year from today. The current spot price is \$300/oz. The one year interest rate is 12%.

a. Compute the theoretical futures price for delivery one year hence.

b. Suppose that the actual futures price equals the theoretical futures price that you calculated in part **a**. The gold you produce is of the quality that is deliverable into the futures contract. Regressing changes in the spot price of the gold you produce on futures price changes would give a slope coefficient of 1.0. If you did not tail your hedge, how many gold futures contracts would you trade? If you tailed your hedge, how many would you trade? Is this a long hedge or a short hedge?

c. Suppose you did not tail the hedge. You borrow to pay daily resettlement cash outflows, and lend daily resettlement cash inflows. One month after the hedge is put on, the spot price of gold rises to \$350/oz. and the futures price rises to \$392/oz. Then, during the remaining 11 months of the hedge, the futures price remains constant, and on the hedge-lifting date, $S_T = F_T =$ \$392/oz. You sell your gold. After accounting for the impact of interest on the daily resettlement that occurred at t = one month, what was your effective selling price of the gold? In other words, sell your gold, add/subtract futures profits/losses, and account for the interest expense and/or interest income.

d. Now, redo part **c** with the tailed hedge.

7.10 You own 100 oz. of gold. Its spot price today is \$300/oz. You wish to protect your investment against a decline in the price of gold. Therefore, do you buy or sell one gold futures contract? Today's gold futures price for delivery one month hence is \$310/oz. Are these prices an example of a normal or an inverted market? Compute the basis. On the delivery date, what will the basis be? Tomorrow, the gold futures price settles at \$304/oz. After marking to market, what is the value of the gold futures contract? Suppose that two weeks from today, you lift the hedge by offsetting the futures contract and selling the gold. On that date, the spot price of gold is \$285/oz. and the futures price is \$297/oz. Compute the total profit or loss on the hedged position, as we did in Table 7.3.

7.11 A mutual fund prospectus states that the managers use S&P 500 futures "for short-term cash management purposes,... to reallocate the fund's assets among stocks while minimizing transaction costs, maintain cash reserves while simulating full investment, facilitate trading, seek higher investment returns, or simulate full investment when a futures contract is priced more attractively or is otherwise considered more advantageous than the underlying securities." Discuss the benefits to which the prospectus is referring.

7.12 Explain why short hedgers are said to be "long the basis" while long hedgers are said to be "short the basis".

7.13 Explain what is meant by the following statement: Basis equals the net cost of carry.

7.14 Input the price change data shown in Table 7.2 into an Excel spreadsheet. Compute (verify) the mean, variance, and standard deviations of the two time series. Then use Excel's regression tool (tool/data analysis/regression) to regress the change in the spot price on the change in the futures price. Find the y-intercept and slope coefficient of the regression model.

CHAPTER 8

Stock Index Futures

Stock index futures contracts began trading on February 24, 1982, when the Kansas City Board of Trade introduced futures on the Value Line Index. About two months later, the Chicago Mercantile Exchange introduced futures contracts on the S&P 500 Index. By 1986, the S&P 500 futures contract had become the second most actively traded futures contract in the world, with over 19.5 million contracts traded in that year.[1] In May 1982, the NYSE Composite Index futures contract began trading on the New York Futures Exchange. In July 1984, the Chicago Board of Trade, frustrated because Dow Jones & Company went to court to block its attempts to trade futures on the Dow Jones Industrial Average (DJIA), finally gave up and began trading futures contracts on the Major Market Index (MMI). The MMI was very similar to the DJIA. Finally, in June 1997, Dow Jones agreed to allow DJIA options, futures, and options on futures to begin trading. On October 6, 1997, futures on the DJIA began trading on the Chicago Board of Trade.

In their short history of trading, stock index futures contracts have had a great impact on the world's equity markets. Trading in stock index futures has allegedly made the world's stock markets more volatile than ever before. Critics also claim that individual investors have been driven out of the equity markets because institutional traders' actions in both the spot and futures markets cause stock values to gyrate with no links to their fundamental values. Many political figures have called for greater regulation, going so far as to favor an outright ban on stock index futures trading. Fortunately, such extreme measures have been avoided. Stock index futures have become irreplaceable in our modern world of institutional money management. They have revolutionized the art and science of equity portfolio management as practiced by mutual funds, pension plans, endowments, insurance companies, and other money managers.

A futures contract on a stock market index represents the right and obligation to buy or sell a portfolio of stocks characterized by the index. Stock index futures are cash settled. Thus, there is no delivery of the underlying stocks. The contracts are marked to market daily, and the futures price is set equal to the spot index level on the last trading day, leaving one last mark-to-market cash flow.

Figure 6.5 showed only some of the wide range of stock index futures contracts that trade globally, including those on the U.S., Japanese, French, British, German, and Australian stock markets. The most actively traded contract is the S&P 500 futures contract, traded on the CME.

We begin this chapter with a discussion of how different indexes are computed.

8.1 What Is An Index?

An index is, in one sense, just a number that is computed to allow measurement of the value of a portfolio of stocks. Other indexes have been constructed to track the values of securities of other types, such as bonds and futures. Still other indexes track such economic indicators as the consumer price index (CPI) or the index of leading indicators. Note in Figure 6.5 that futures contracts trade on the GSCI (the Goldman Sachs Commodity Index, which is an index of commodity prices), and the U.S. Dollar Index (which is an average value of the dollar).

This section describes three different stock market indexes. When one is constructing a stock market index, three issues are of particular interest: which stocks are in the index, how each stock is weighted, and how the average is computed.

8.1.1 Price-Weighted Indexes: The Dow Jones Average

From 1982 through 1984, the CBOT tried unsuccessfully to trade futures contracts on the Dow Jones Industrial Average (DJIA).[2] As just mentioned, however, Dow, Jones & Company sued to block the efforts of the CBOT, and won in court. It was not until October 1997 that Dow Jones permitted futures trading on the DJIA, which is the most widely followed index by the investing public. The question "How is the market doing?" is usually assumed to refer to the DJIA. Other stock market indices that are computed in the same way as the DJIA, and which have futures and options trading on them include the Nikkei 225 stock index of Japanese stocks.

As of December 2001, the 30 stocks in the DJIA were those in Table 8.1. The DJIA stocks are large corporations. Fourteen of the companies were among the largest 39 companies in the world, according to August 15, 2000, market capitalizations.

The DJIA is computed by adding the prices of the 30 component stocks and dividing the sum by a divisor that is printed in the *Wall Street Journal* every day and is also available in the equity product information area of the CBOT website (www.cbot.com). For example, on April 24, 2001, the divisor for the DJIA was 0.15369402. On September 9, 1999, the divisor was 0.19740463. The divisor is changed when one of two events occurs. Periodically, one of the 30 stocks in the DJIA is removed and replaced by the stock of another company. This will happen when a component stock is taken over by another company or one of the corporations goes bankrupt. For example,

TABLE 8.1 The 30 Stocks in the Dow Jones Industrial Average as of December 2001

Alcoa	Honeywell
American Express	IBM
AT&T	Intel
Boeing	International Paper
Caterpillar	Johnson & Johnson
Citigroup	McDonald's
Coca Cola	Merck
Disney	Microsoft
DuPont	Minnesota Mining & Manufacturing (MMM)
Eastman Kodak	J.P. Morgan Chase
Exxon Mobil	Philip Morris
General Electric	Procter & Gamble
General Motors	SBC Communications
Hewlett Packard	United Technologies
Home Depot	Wal-Mart

Anaconda, long a component of the DJIA, was bought by Atlantic Richfield (ARCO) in 1976. Thus, a new component stock (MMM) was selected to replace Anaconda. At other times, it is decided that the composition of the index stocks is no longer representative of "the market," and Dow Jones deletes one or more stocks, and adds others. This occurred on November 1, 1999, when Home Depot, Intel, Microsoft, and SBC Communications replaced Chevron, Goodyear, Sears Roebuck, and Union Carbide.

The DJIA divisor is also adjusted to reflect stock splits and stock dividends. These events reduce stock prices; without some sort of adjustment in the method of computation, the DJIA would most likely drop sharply every time one of its component stocks split its shares.

The DJIA is called a "price-weighted" index because the impact of each component stock is proportional to its stock price. We now illustrate how a price-weighted average like the DJIA is computed, and how it adjusts for events that affect the divisor. Suppose there were only three stocks in the DJIA. On the first day, the divisor is 2.6, and the prices of the three stocks are:

Stock	Price (Day 1)
A	19 5/8
B	27
C	52 1/2

The DJIA would be computed to be $(19.625+27+52.5)/2.6=38.125$. At the close of trading on day 1, suppose company A goes bankrupt or is taken over by another firm. Dow Jones decides to replace it with company D, which closed at \$39/share. To maintain continuity of the index, the divisor is changed so that the sum of the three new prices, divided by the new divisor, equals 38.125. The value of the new divisor is found by solving the following equation for X: $(39+27+52.5)/X=38.125$. The new divisor would be $X=3.108197$.

Now, suppose the next day (day 2), the prices of the component stocks are:

Stock	Price (Day 2)
D	39 1/8
B	27 1/2
C	51 1/4

The DJIA would be computed to be $(39.125+27.5+51.25)/3.108197=37.924$. The following day (day 3), suppose stock C splits 3 for 2, and the three component stocks close as follows:

Stock	Price (Day 3)
D	40
B	28 1/8
C	34 1/2

The index value must be preserved against the effects of the split, and this is accomplished by changing the divisor. Stock C's postsplit price equivalent to the presplit price of \$51.25 is $(51.25/1.5=)$ 34.1667. Thus, the new divisor would become the value of X in the equation $(39.125+27.5+34.16667)/X=37.924$. The new divisor is $X=2.65773$. Stock C's closing price on day 2 has been adjusted to reflect the 3-for-2 split, and then a new divisor is calculated to preserve day 2's index value of 37.924. Now, the DJIA on the ex-split day (day 3) can be computed to be $(40+28.125+34.5)/2.65773=38.614$.

The DJIA is not adjusted to account for regular dividend payments. On some days several component stocks of the DJIA trade ex-dividend. Because each stock will open lower by about the dividend amount, the DJIA will open lower by an amount approximately equal to the sum of the dividends per share divided by the divisor.

To construct a portfolio that is equivalent to the DJIA, an investor must buy an equal number of shares of each of the component stocks. Maintaining the proper underlying portfolio is complicated by the payment of cash dividends and stock distributions. Still, the DJIA is an easy index to replicate, since it has only 30 stocks, each of which is very actively traded.

Because of the weighting and averaging scheme, high-priced stocks carry more weight than low-priced stocks in affecting the movements of price-weighted indexes. If a stock selling for $100/share increases in value by 5%, then the DJIA will increase by 5/divisor points. If a stock selling for $20/share rises in price by 5%, then the DJIA will rise by only 1/divisor point. In other words, any percentage change in price by a high-priced stock will have a greater impact than the same percentage change in price for a low-priced stock.

Futures on the DJIA trade on the Chicago Board of Trade. As shown in Figure 6.5, the value of stock underlying one DJIA futures contract equals $10 times the futures price. One tick is one Dow-point, and this equals $10. Thus, if the DJIA futures price rises one tick, from 10813 to 10814, a trader who is long one contract profits by $10 because the value of the stock underlying the contract rises from $108,130 to $108,140.

8.1.2 Value-Weighted Averages: The S&P 500 Stock Index

Many stock market indexes are value-weighted averages. Most academics would likely argue that a value-weighted index is the best measure for market performance. In the capital asset pricing model (CAPM), a stock's correlation with the market portfolio is the factor that determines its price, and that market portfolio is value weighted.

Besides the NYSE and S&P Indexes, other value-weighted indexes include the Amex Market Value Index, the NASDAQ Composite Index, the Russell 2000 Index, and the Wilshire 5000. The levels of these and yet other indexes are presented daily in the *Wall Street Journal.* While each index is a different portfolio of stocks, the method of computing each index is the same.

To compute the level of a value-weighted index, first find the market value of each of the component stocks on a base day, day 0. The market value of stock i, MV_i, is computed by multiplying the price of stock i, P_i, by the number of shares outstanding of stock i, N_i.

$$\beta_d = w_1 b_1 + w_2 b_2$$

Next, find the total market value of all of the component stocks of the index. That is, sum the market values of all of the component stocks on day 0.

$$MV_0 = MV_{0,1} + MV_{0,2} + MV_{0,3} + \cdots + MV_{0,n}$$

$$MV_0 = \sum_{j=1}^{n} MV_{0,j}$$

Once the total market value of all the n component stocks on day 0 has been found, divide that total market value by an arbitrary divisor to set the initial index value. Call that value I_0. Then, to find the index value on any subsequent day, day t, find the ratio of market values on days t and $t-1$, and multiply the ratio by the earlier day's index value, I_{t-1}.

$$I_t = \frac{MV_t}{MV_{t-1}} \times I_{t-1}$$

EXAMPLE 8.1 Consider the following data on prices and shares outstanding for the four component stocks of an index:

Stock	Shares Outstanding	P_0	P_1	P_2
A	1000	50	52	49
B	1000	10	10	11
C	100	60	60	63
D	100	10	11	12

On the base day, day 0, the total market value of the four component stocks is:

$$MV_0 = (50)(1000) + (10)(1000) + (60)(100) + (10)(100) = 67,000$$

The index developer decides that 100 ought to be the initial index level, so the divisor is set at 670. To compute the index level on day 1, first find and sum the market values of the component stocks:

$$MV_1 = (52)(1000) + (10)(1000) + (60)(100) + (11)(100) = 69100$$

Then multiply the ratio of the two days' total market values by day 0's index value:

$$I_1 = \frac{MV_1}{MV_{01}} \times I_0 = \frac{69,100}{67,000} \times 100 = 103.134$$

The index level on day 2 is found in one of two ways:

$$I_2 = \frac{MV_2}{MV_0} \times I_0 = \frac{67,500}{67,000} \times 100 \quad \text{or} \quad \frac{MV_2}{MV_1} \times I_1 = \frac{67,500}{69,100} \times 103.134 = 100.746$$

Note that even though three of the component stocks rose in price on day 2, the market value index fell. This is because stock A is the largest stock, in terms of market value. Changes in the value of stock A carry the most weight in computing changes in the index level.

As of December 19, 2001, the four largest U.S. firms, as measured by the market values of their equity, were (1) General Electric (GE; market value = \$394.3 billion), (2) Microsoft (MSFT; market value = \$373.1 billion) (3) Exxon Mobil Corp (Xom; market value = \$256.9 billion), and (4) Pfizer (PFE; market value = \$255.5 billion). All these stocks are in the S&P 500 and S&P 100 Indexes. Together, just these four stocks made up about 12% of the market value of all of the S&P 500 listed stocks. A 1% change in the price of GE would have a 10 times higher impact on a value-weighted index than a 1% change in the value of DaimlerChrysler, which had a market value of \$41.2 billion on December 19, 2001. Some investors attempt (risky) arbitrage by trading portfolios of only 20 to 50 stocks and S&P derivatives. The reasoning is that just 50 stocks might make up 50% of the index and that the correlation of the 50-stock portfolio will therefore correlate very highly with the underlying stock index. However, these investors are bearing a form of basis risk when they rely on such a strategy. The total market value of the component stocks of the S&P 500 together represent about 70% of the market value of all U.S. equities.

Stock splits and stock dividends, in principle, do not affect the market values of companies. Thus, there are no adjustments needed to the method of computing a value-weighted index when one of the component stocks has a stock distribution. Like the DJIA, the computation of these value-weighted indexes is not affected by the payment of regular cash dividends. An investor who replicated a portfolio characterized by a value-weighted index would profit by the percentage change in the index (capital gains) and also by dividends paid. However, dividends would not show up in the index value.

Adjustments occur when the "owner" of the index removes companies from the portfolio, either for arbitrary reasons or because of mergers or bankruptcies. For example, Standard & Poor's periodically revises the stocks that make up the S&P 100 and S&P 500 Indexes to make these listings more representative of the market. Also, when firms make changes in their equity capitalization by issuing or repurchasing stock, adjustments are made.[3]

To form a portfolio of stocks that would replicate a market-value-weighted index, an investor must purchase $x\%$ of the market value of each component stock, where x is some arbitrary constant. For example, one could buy 0.3% of each of stocks A, B, C, and D in Example 8.1. Given their initial market values of $50,000, $10,000, $6000, and $1000, respectively, the investor would invest $150 in stock A, $30 in stock B, $18 in stock C, and $3 in stock D.

Many futures trade on value-weighted indexes. Trading on the CME alone are the S&P 500 futures (the underlying stock is $250 times the index), mini-S&P 500 futures (the underlying stock is only $50 times the index), as well as contracts on the S&P Midcap 400, NASDAQ 100, Russell 2000 indexes, and others. The Midcap 400 is an index of 400 middle-sized firms, while the NASDAQ 100 tracks the performance of the over-the-counter stock market in the United States. The Russell 2000 index is usually used to measure the performance of the asset class known as small cap stocks. The broadest value-weighted index is the Wilshire 5000 index. It consists of every NYSE- and Amex-listed stock, and all actively traded OTC stocks. However, neither options nor futures trade on the Wilshire 5000 index. At times, investors believe that it may be more profitable to concentrate a portfolio in either growth or value stocks. Standard & Poor's Corporation and BARRA, Inc., jointly formed the S&P 500/BARRA Growth and Value Indexes. Each company in the S&P 500 Index is assigned either to the Growth or the Value Index. The indices are designed with about 50% of the S&P 500 capitalization in the Value Index and 50% in the Growth Index.[4]

S&P 500 stock index futures contracts are perhaps the most actively traded stock index futures in the world. The last trading day for this contract is the Thursday before the third Friday of the delivery month, and there are four delivery months, March, June, September, and December. The smallest price change is 0.10 point,[5] which equals $25. Thus, if the S&P 500 futures price falls from 1419.40 to 1419.30, the value of the stock underlying the contract declines from (1419.40×250=) $354,850 to (1419.30×250=) $354,825. This one-tick change in the futures price creates a mark to market profit of $25 for an individual who is short one contract.[6]

8.1.3 The Value Line Index

Futures contracts on the Value Line Index trade on the Kansas City Board of Trade. The Value Line Index is an equal-weighted arithmetic average.[7] To compute it, pick any arbitrary index value on the base date, day 0. The index value on any subsequent day, day t, is then:

$$I_t = I_{t-1}\frac{(P_{1,t}/P_{1,t-1})+(P_{2,t}/P_{2,t-1})+(P_{3,t}/P_{3,t-1})+\cdots+(P_{n,t}/P_{n,t-1})}{n}$$

In other words, the index value on day t is the arithmetic average of the price relatives of the n stock, times the preceding day's index value. There are about 1700 stocks in the Value Line Index. They are the stocks followed by the Value Line Investment Service, and include several AMEX and OTC stocks, and even some stocks that trade on Canadian and regional stock exchanges.

To illustrate the calculation of an index like the Value Line Index, consider the data from Example 8.1 of this chapter:

Stock	P_0	P_1	P_2
A	50	52	49
B	10	10	11
C	60	60	63
D	10	11	12

The initial index value on day 0 is arbitrarily set at 50. On day 1, the index is

$$I_1 = \frac{50(52/50 + 10/10 + 60/60 + 11/10)}{4} = \frac{50(4.14)}{4} = 51.75$$

On day 2, the index value is:

$$I_2 = \frac{51.75(49/52 + 11/10 + 63/60 + 12/11)}{4} = \frac{51.75(4.1832)}{4} = 54.1204$$

To replicate an equally weighted arithmetically averaged index like the Value Line, invest an equal number of dollars in each of the component stocks.

EXAMPLE 8.2 The accompanying table gives a 4-day time series for each of three stocks. Compute the index values if they were patterned after (1) a price-weighted arithmetic index (such as the DJIA), (2) a value-weighted arithmetic index (such as the S&P 500), and (3) an equal-weighted arithmetic average (such as the Value Line Index). You can compute the necessary divisors from the index values on day zero. How many shares of each stock would you purchase to replicate each index?

	Stocks			Indexes		
Day	**A**	**B**	**C**	**1**	**2**	**3**
0	10	50	100	10	10	10
1	11	52	99			
2	12	55	95			
3	15	54	98			
4	10	50	95			
Shares outstanding	1000	20	100			

The solution to this problem is as follows. As can be seen from the resulting index values, the method by which an index is created can lead to different pictures of to how the market is performing.

	Stocks			Indexes		
Day	**A**	**B**	**C**	**1**	**2**	**3**
1	11	52	99	10.125	10.4476	10.4333
2	12	55	95	10.125	10.7619	10.8096
3	15	54	98	10.4375	12.3238	11.7586
4	10	50	95	9.6875	9.7619	10.0418

- **DJIA (index 1)**: On day 0, the sum of the prices ΣP_i is 160. The index value is 10, so the divisor is 16. On each subsequent day, add the prices and divide by 16.
- **S&P Indexes (index 2)**: On day 0 the sum of the market values is (10,000+1000+10,000=) 21,000. The index on day 0 is 10, so the divisor is 210. On each subsequent day, multiply:

$$\frac{MV_t}{MV_0} \times I_0$$

  That is, divide the market value of all stocks on day t by the market value on day 0 and multiply by the day 0 index value of 10.
- **Value Line Index (index 3)**: Set the day 0 index value to 10. On each subsequent day, find the arithmetic mean of the price relatives and multiply it by the previous day's index. So, on day 1:

$$\frac{11/10+52/50+99/100}{3} \times 10 = \frac{3.13}{3} \times 10 = 10.4333$$

To replicate the rates of return on index 1, one would purchase an equal number of shares of each of the component stocks. For example, on day 0, buy one share each of stock A, B, and C. This will cost $160. On day 1, the value of the portfolio is $162. The rate of return is (162 − 160)/160 = 1.25%, which equals the rate of return on the index [(10.125 − 10)/10 = 1.25%].

To replicate index 2, one would invest an amount in each stock that is proportional to its market value. For example on day 0, the market values of stocks A, B, and C are $10,000, $1000, and $10,000, respectively. The total market value is $21,000. Proportionally, the market values of A, B, and C are 0.4761905 (= 10,000/21,000), 0.047619 (= 1000/21,000), and 0.4761905, respectively. If you had $100 to invest, you would invest $47.61905 in stocks A and C, and $4.7619 in stock B. Therefore, on day 0, purchase 4.761905 shares of stock A at $10/share, 0.095238 share of stock B at $50/share, and 0.4761905 share of stock C at $100/share.

To replicate index 3, one would invest the same dollar amount in each of the three stocks. If you had $99 to invest, you would invest $33 in each stock. This means that you would buy 3.333 shares of stock A, 0.66 shares of stock B, and 0.33 shares of stock C.

Note that index 3, the equally weighted arithmetically averaged index (i.e., the Value Line) is the most costly to replicate because every day the dollar amounts invested in each stock must be readjusted so that they are again equal. No such rebalancing is necessary for the other two indexes.

8.2 PRICING STOCK INDEX FUTURES

The fundamental futures and forwards pricing equation, from Chapter 5, is the cost-of-carry model:

$$F = S + CC - CR \tag{8.1}$$

futures price = spot price + carry costs − carry return

For stock index futures contracts, the spot price is the spot index value.[8] The carry cost represents the interest on the value of the stock underlying the cash portfolio; it equals the interest borrowing cost when one is performing cash-and-carry arbitrage and the interest earned on lending the proceeds from selling of stock in the case of reverse cash-and-carry arbitrage. The carry return is found by calculating the future value of the dividends received between today and the delivery date, if the portfolio of stocks underlying the index is owned. In other words, the dividends received are invested to earn interest, from the day they are received until the delivery date.[9] Summarizing, we write

$$F = S + S[h(0,T)] - \Sigma[FV(\text{divs})] \tag{8.2}$$

where $h(0,T) = rT/365$, where r is the annual interest rate, T is the number of days until the delivery date, and $\Sigma(FV(\text{divs}))$ is the future value of all dividends paid by the component stocks of the index.

Proof

Suppose: $F_0 > S_0 + S_0[h(0,T)] - \Sigma[FV(\text{divs})]$
Then, $F_0 - S_0 - S_0[h(0,T)] + \Sigma[FV(\text{divs})] > 0$
Or: $F_0 - S_0[1 + h(0,T)] + \Sigma[FV(\text{divs})] > 0$

Today

Buy the component stocks of the index in the appropriate weights, so that the index is replicated	$-S_0$
Borrow the foregoing amount for the number of days until the delivery date, at an annual interest rate of $r\%$, which is $h\%$ unannualized	$+S_0$
Sell one futures contract at its current futures price of F_0 (no cash flow results; use a bank letter of credit, or securities, to meet initial margin requirements)	0
Total cash flow	0

On the first ex-dividend date of one or more stocks, t_1

Receive dividends	+divs
Lend dividends[10] from t_1 to T	−divs
Total cash flow	0

On subsequent ex-dividend dates of component stocks (i.e., on days $t_2, t_3, \ldots, T$)

Receive dividends	$+\text{divs}$
Lend dividends from $t_2, t_3, \ldots, T$ until time T	$-\text{divs}$
Total cash flow	0

On the delivery date

Buy (offset) the futures contract at F_T	$+F_0 - F_T = +F_0 - S_T$
Sell the stocks	$+S_T$
Repay loan and interest	$-S_0 - h(0, T)S_0$
Receive dividends, and interest on those dividends	$+\Sigma FV(\text{divs}) = +\text{divs}_{t1}[1 + h(t1, T)]$ $+\text{divs}_{t2}[1 + h(t2, T)] \cdots + \cdots$ $+\text{divs}_{T-1}[1 + h(T-1, T)] + \text{divs}_T$
Total cash flow	$+F_0 - S_0 - h(0, T)S_0 + \Sigma FV(\text{divs}) > 0$

Therefore, if market participants observe situations in which $F_0 - S_0[1 + h(0, T)] + \Sigma(FV(\text{divs})) > 0$, they will engage in a cash-and-carry arbitrage. The futures price is "overpriced" in terms of the spot index. Therefore, the cash-and-carry arbitrage calls for buying the spot index (it is relatively cheap), "carrying" it, and selling the futures (it is relatively expensive).

Proof of the other inequality [that arbitrage opportunities exist if $+F_0 - S_0 - h(0, T)S_0 + \Sigma FV(\text{divs}) < 0$] was essentially presented in Chapter 5. You should recall that the set of reverse cash-and-carry trades requires selling stocks, lending the proceeds, and buying futures. To sell stocks requires either selling stock short (in which case we have "pure arbitrage"), or selling stocks already owned (in which case we have "quasi-arbitrage"). In either case, we assume that the arbitrageur will receive full use of the proceeds from the short sale and will not have to post any margin.

If market participants observe situations where $F_0 - S_0(1 + h(0, T)) + \Sigma(FV(\text{divs})) < 0$, they will engage in a reverse cash-and-carry arbitrage. When this occurs, we say that "selling programs have hit the market," and stock prices will almost surely decline. The futures price is "underpriced" in terms of the spot index. Therefore, the reverse cash-and-carry arbitrage calls for buying the futures (it is relatively cheap), selling the spot (it is relatively expensive), and investing the proceeds.

8.2.1 An Illustration of Stock Index Futures Pricing

EXAMPLE 8.3 Consider the D&M3, a value-weighted index of three stocks. Today, at time 0, their prices, shares outstanding, and market values are:

Stock	Price	Shares Outstanding	Market Value
1. Danish Wine Imports	\$40	0.25 million	\$10 million
2. Maxfu Enterprises, Inc.	\$30	0.667 million	\$20 million
3. Miss Molly.com	\$25	2 million	\$50 million

In terms of these market values, the composition of the D&M3 index is $^1/_8$ of stock 1, $^1/_4$ of stock 2, and $^5/_8$ of stock 3.

Suppose the spot index is 1350 and the multiplier of the index is 250. Therefore, there is \$337,500 worth of stock underlying the spot index (1350 × 250). Of this total, $^1/_8$, or \$42,187.5, is stock 1. Stock 2 contributes $^1/_4$ of the value of the portfolio, \$84,375. Finally, $^5/_8$ of \$337,500 is \$210,937.50, and this is the value of stock 3 in the aggregate index. Given these values, we can compute the number of shares of each stock that must be bought to replicate the D&M3 index:

Stock	Dollar Value of Stock in 250 Shares of the D&M3	Number of Shares Needed to Replicate 250 Shares of the Index
1	42,187.50	42,187.50/40 = 1,054.6875
2	84,375.00	84,375.00/30 = 2812.50
3	210,937.50	210,937.50/25 = 8437.50

In addition, suppose stock 1 will trade ex-dividend in the amount of \$0.80/share 14 days from today. Stock 2 will trade ex-dividend in the amount of \$0.50/share 55 days from today. Stock 3 pays no dividends. For simplicity, assume that the dividends will be paid on the ex-dates. The riskless interest rate is 5% per year, and will remain there for all maturities at all dates. Given this information, let us find the theoretical futures price if the futures contact expires in 76 days.

Solution

$$h(0,76) = \frac{rT}{365} = \frac{(0.05)(76)}{365} = 0.01041 = 1.041\%$$

The dividend on stock 1 is received 14 days hence and can be invested for 62 days: $h(14,76) = (0.05)(62)/365 = 0.008493$. The dividend on stock 1 will earn an unannualized interest rate of 0.8493% over 62 days. Similarly, the dividend from stock 2 can be invested for 21 days, at an unannualized rate of $h(55,76) = (0.05)(21)/365 = 0.00288$.

Ignoring transactions costs, the theoretical futures price, per share, for the D&M3 futures contract is:

$$F = [(1350)(250) + (1350)(250)(0.01041) \\ -[(1054.6875)(0.80)(1.008493) + (2812.5)(0.50)(1.00288)]]/250$$

$$F = 1355.01$$

If the observed futures price exceeds 1355.01, then arbitrageurs will perform the cash-and-carry arbitrage. They will borrow money to buy the relatively cheap stocks in the D&M3 index and sell the relatively expensive futures contract. If the observed futures price is less than 1355.01, arbitrageurs will reverse these trades. That is, they will sell the stock, invest the proceeds, and buy the futures contract.